MATHEMATICAL BIOLOGY

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ELAM - UFABC - 27/08/2018

SUMMARY

- Modelling approaches
- Theory and scales
- Individuals
- Populations
- Communities
- Epidemics
- More!

MODELLING APPROACHES

To understand the causal relationships at a general level.

Mathematical Modeling

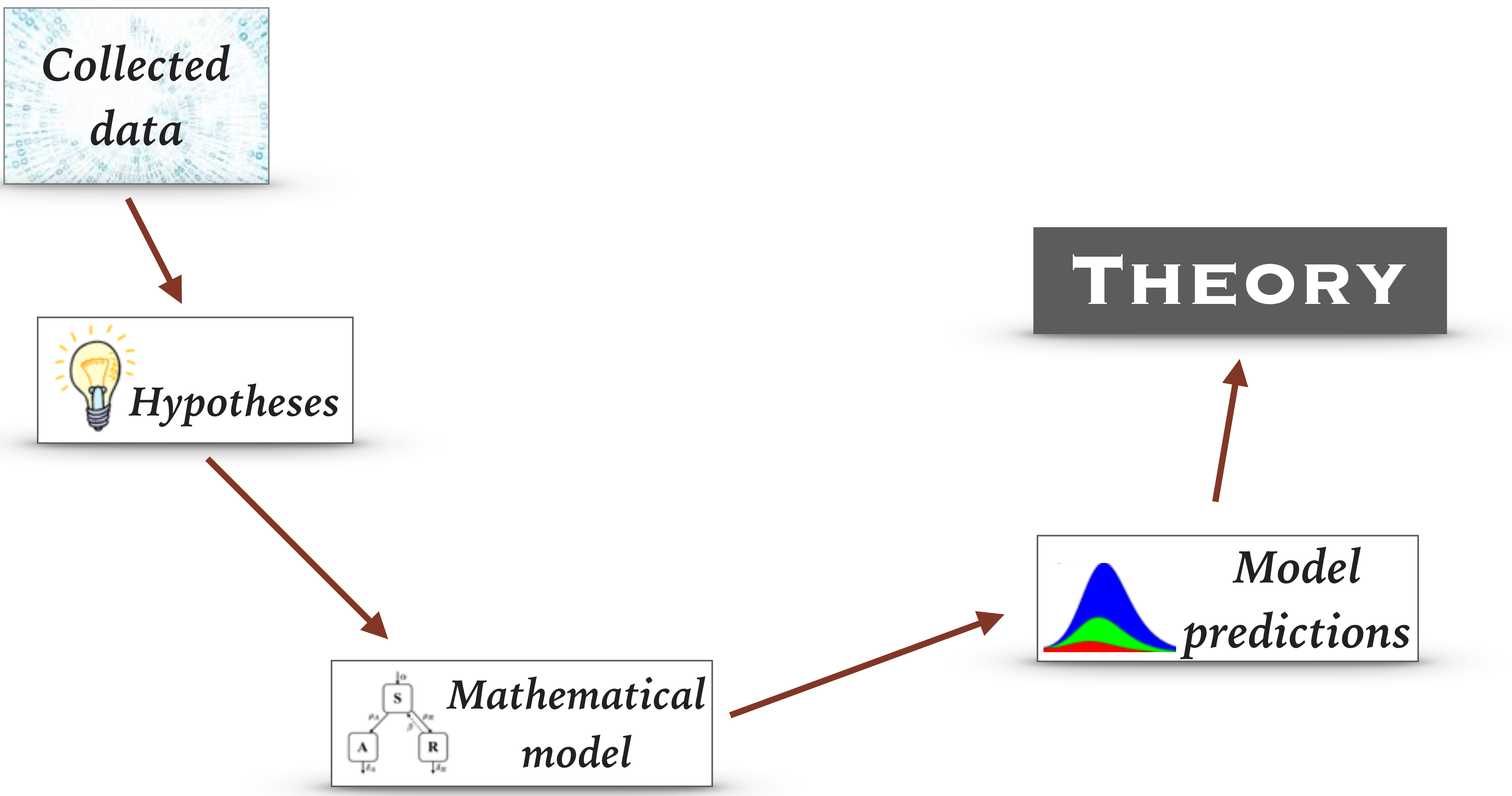
mechanistic

phenomenological

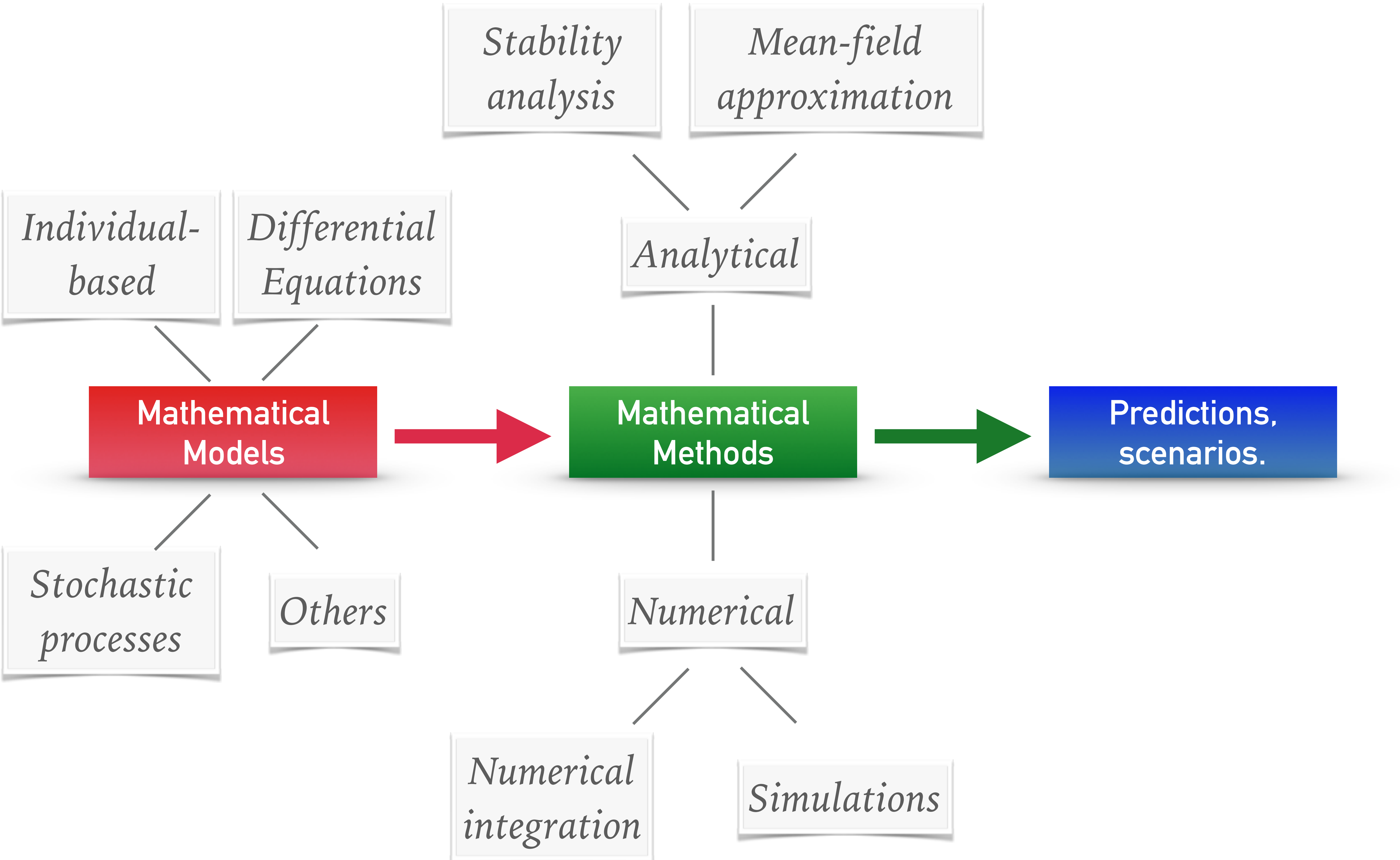
Statistical Modeling

To find factors that shape the empirical data.

MATHEMATICAL MODELING



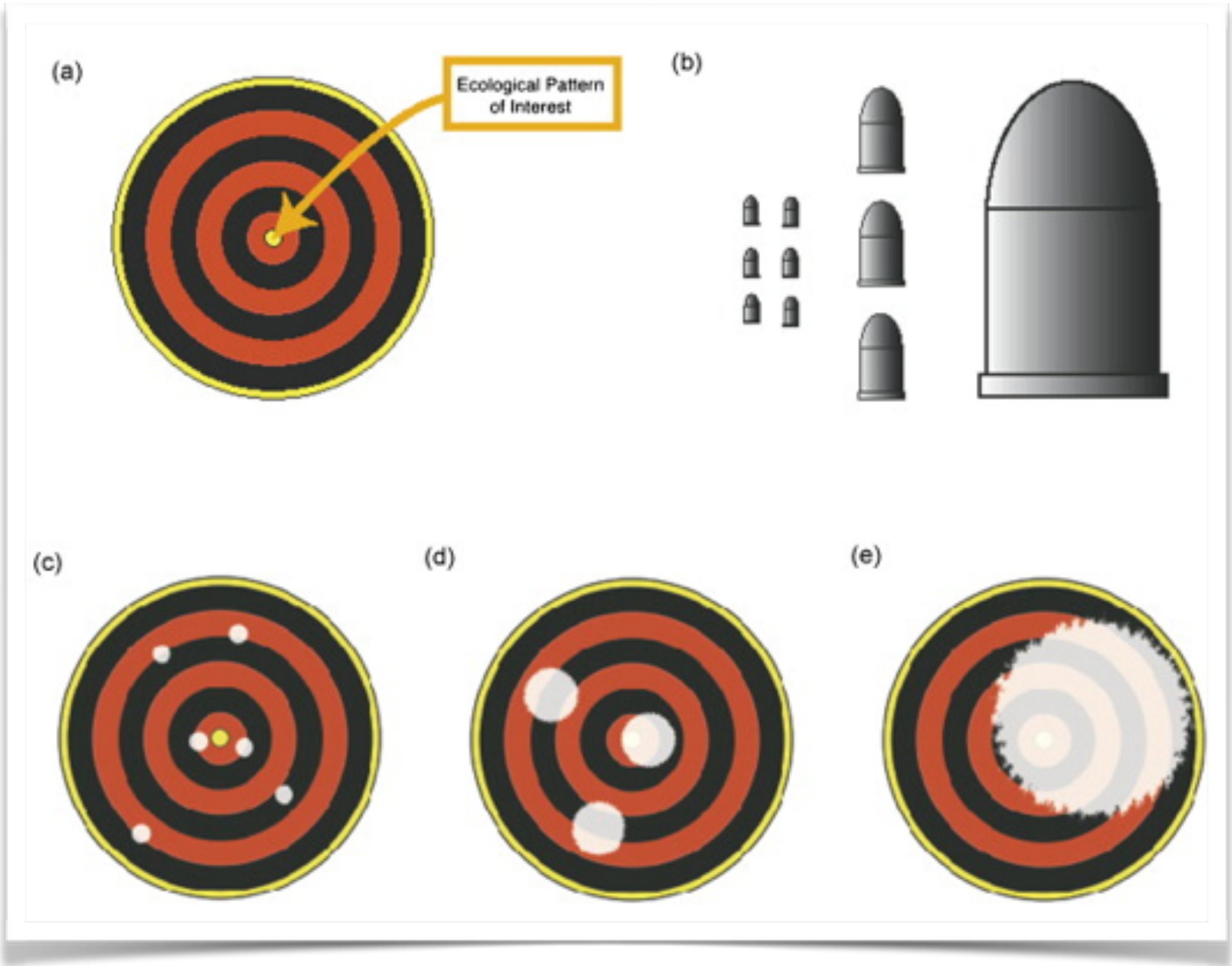
MATHEMATICAL MODELS AND METHODS



MATHEMATICAL MODELS AND METHODS

Variable	Space	Time	Stochasticity	Model type(s)
Discrete	No	Discrete	No	—
			Yes	Markov chain
		Continuous	No	—
			Yes	Markov process
	Discrete	Discrete	No	—
			Yes	Multidimensional Markov chain, discrete-time IBM on grid or patch network, discrete-time SPOM
		Continuous	No	—
			Yes	Multidimensional Markov process, continuous-time IBM on a grid or patch network, continuous-time SPOM
	Continuous	Discrete	No	—
			Yes	Discrete-time individual-based model in continuous space
		Continuous	No	—
			Yes	Spatiotemporal point process
Continuous	No	Discrete	No	Difference equation
			Yes	Stochastic difference equation
		Continuous	No	Differential equation, integral equation
			Yes	Stochastic differential equation
	Discrete	Discrete	No	System of difference equations
			Yes	System of stochastic difference equations
		Continuous	No	System of differential equations
			Yes	System of stochastic differential equations
	Continuous	Discrete	No	Integro-difference equation
			Yes	Stochastic integro-difference equation
		Continuous	No	Partial differential equation, differential equation with convolution
			Yes	Stochastic partial differential equation

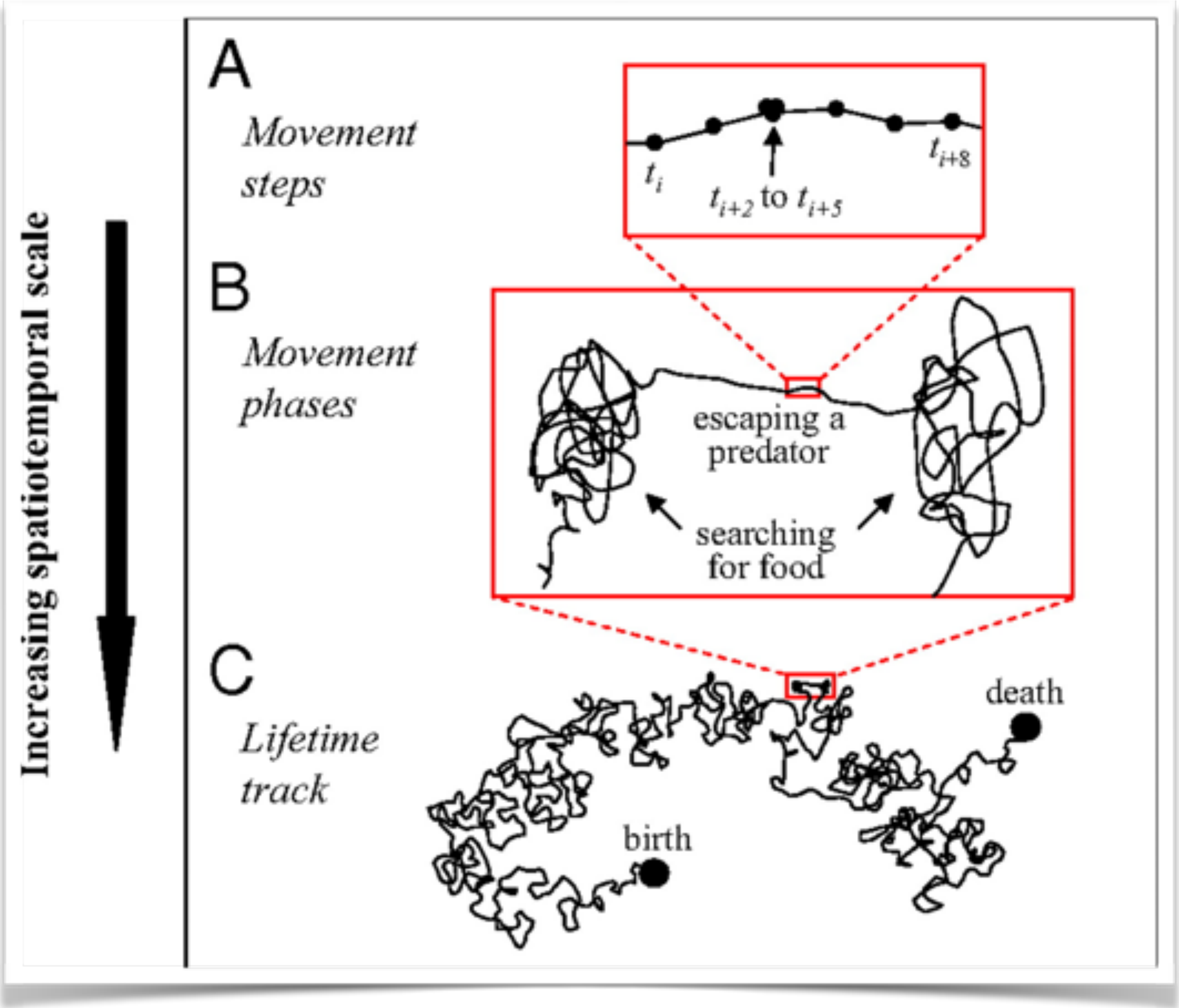
CHOOSING THE PROPER MODEL



- Interpretation of mathematical concepts in favor of biology.
- Identify the level of generality.
- Evaluate the extent to which there is empirical support.

Ginzburg et al. Ecol.Mod. 2007(207)356–362

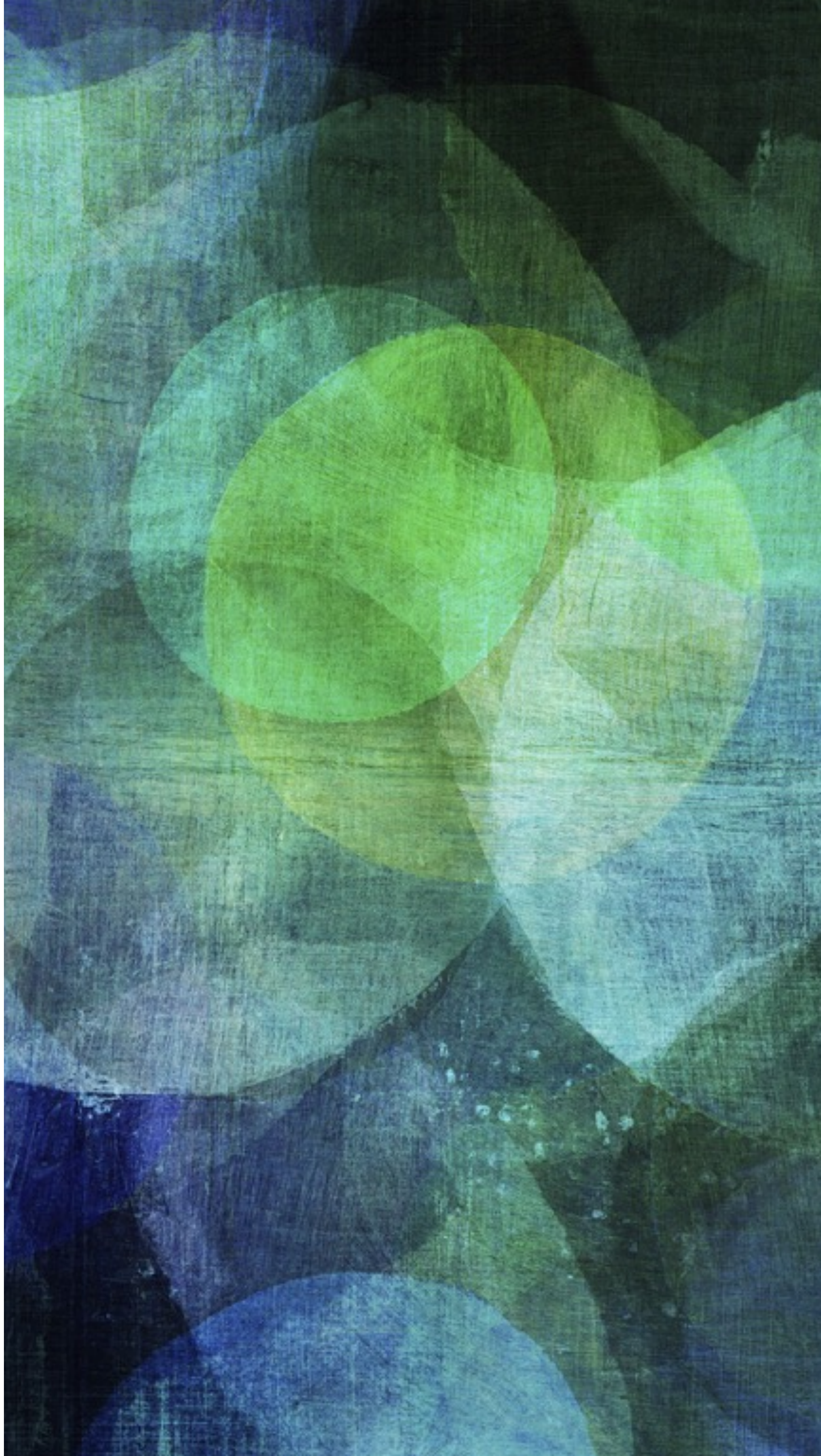
CONSIDER THE SCALE



- Time scale: ecological or evolutionary
- Spatial scale

Ran Nathan et al. PNAS 2008;105:19052-19059

THEORETICAL ECOLOGY EXAMPLES

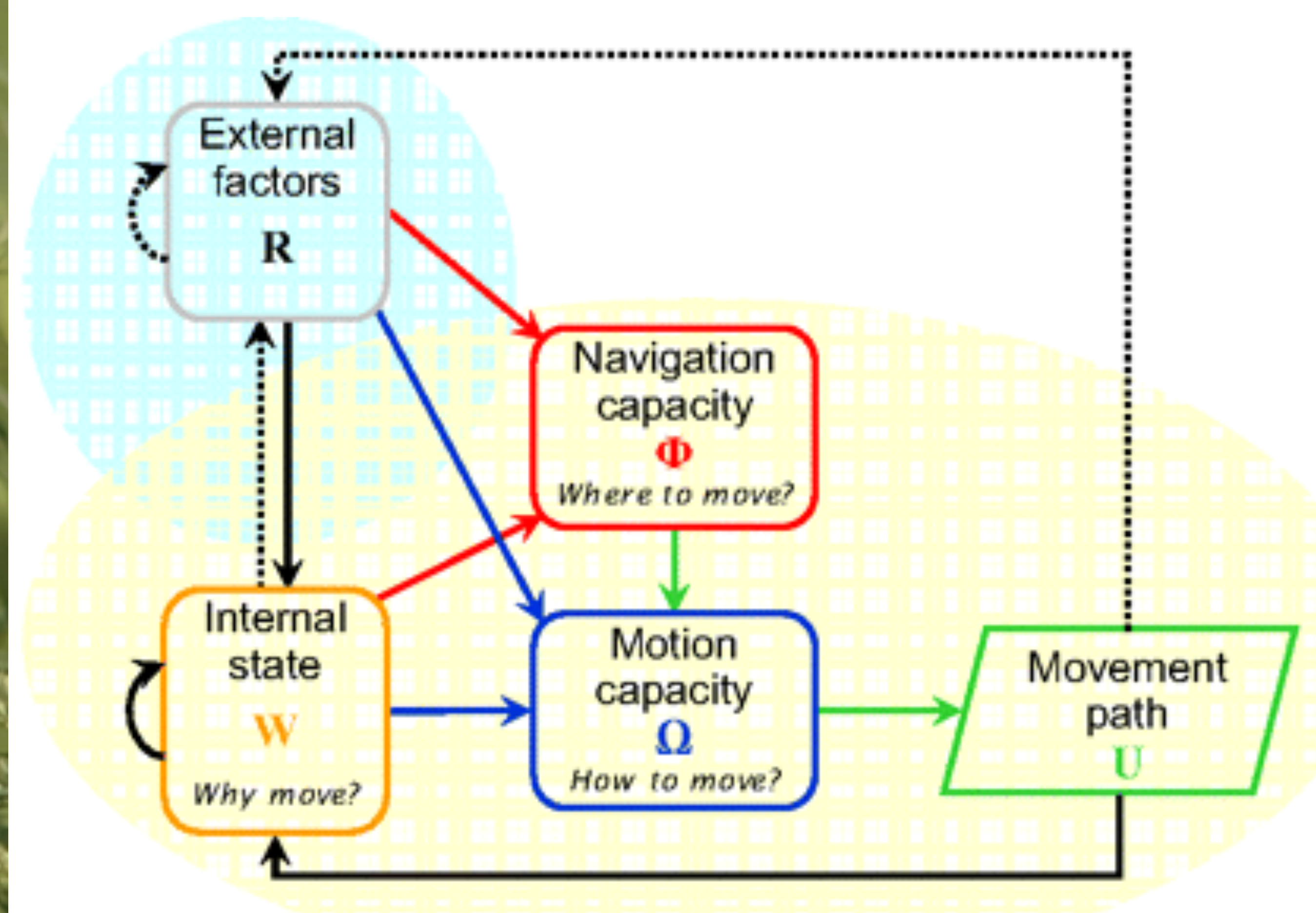


Movement Ecology

INDIVIDUALS

Individuals' movement…

- Why move?
- Where to move?
- How to move?
- When?



Ran Nathan et al. PNAS 2008;105:19052-19059

INDIVIDUALS

Considering observed trajectories:

- Where to move?

Does spatial memory influence the movements decision?

IBM
Simulations

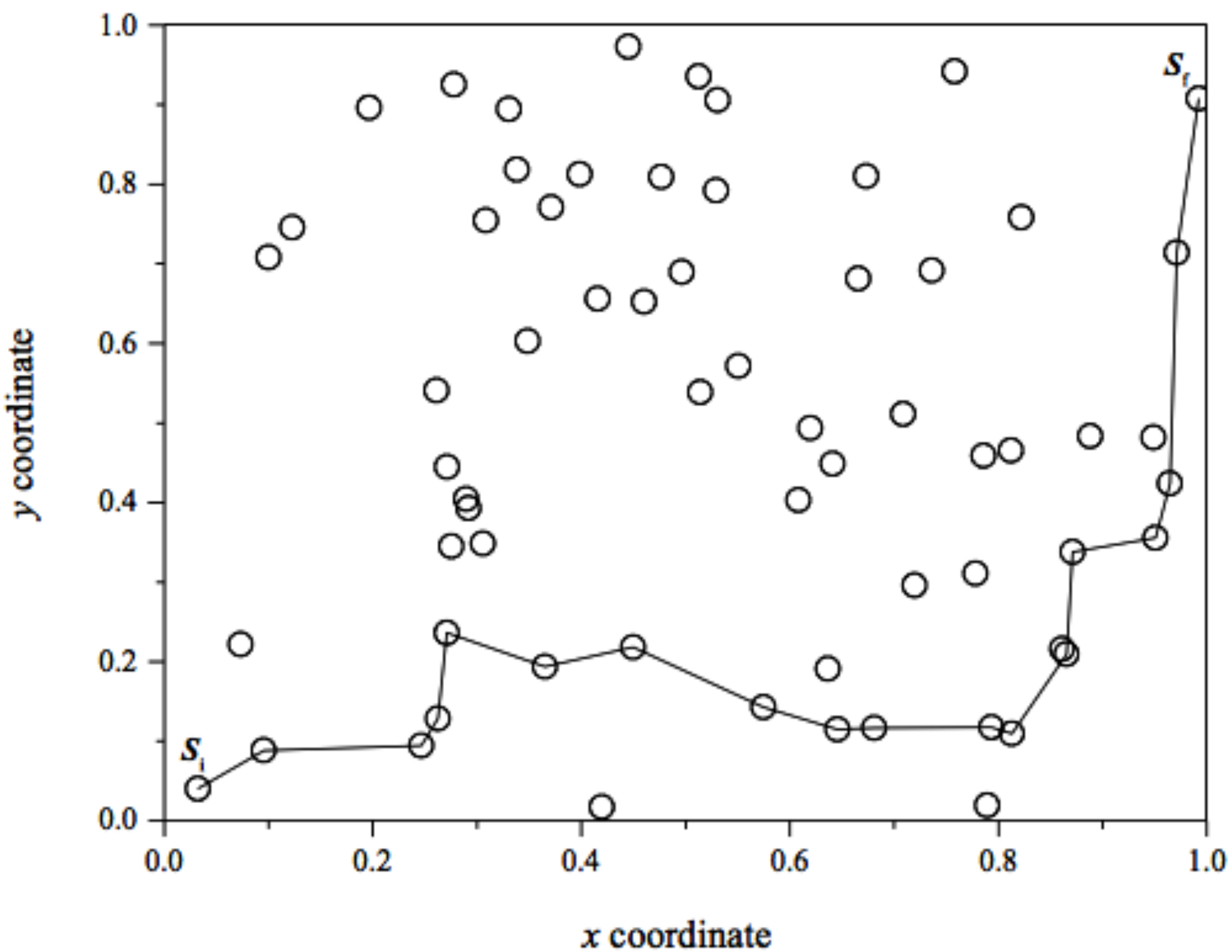
Berbert and Fagan
Ecological Complexity
2012 (12) 1-12.

PDE
Numerical integration

Berbert and Lewis
Ecological Complexity
2018 (33) 41-48.

INDIVIDUAL-BASED MODEL

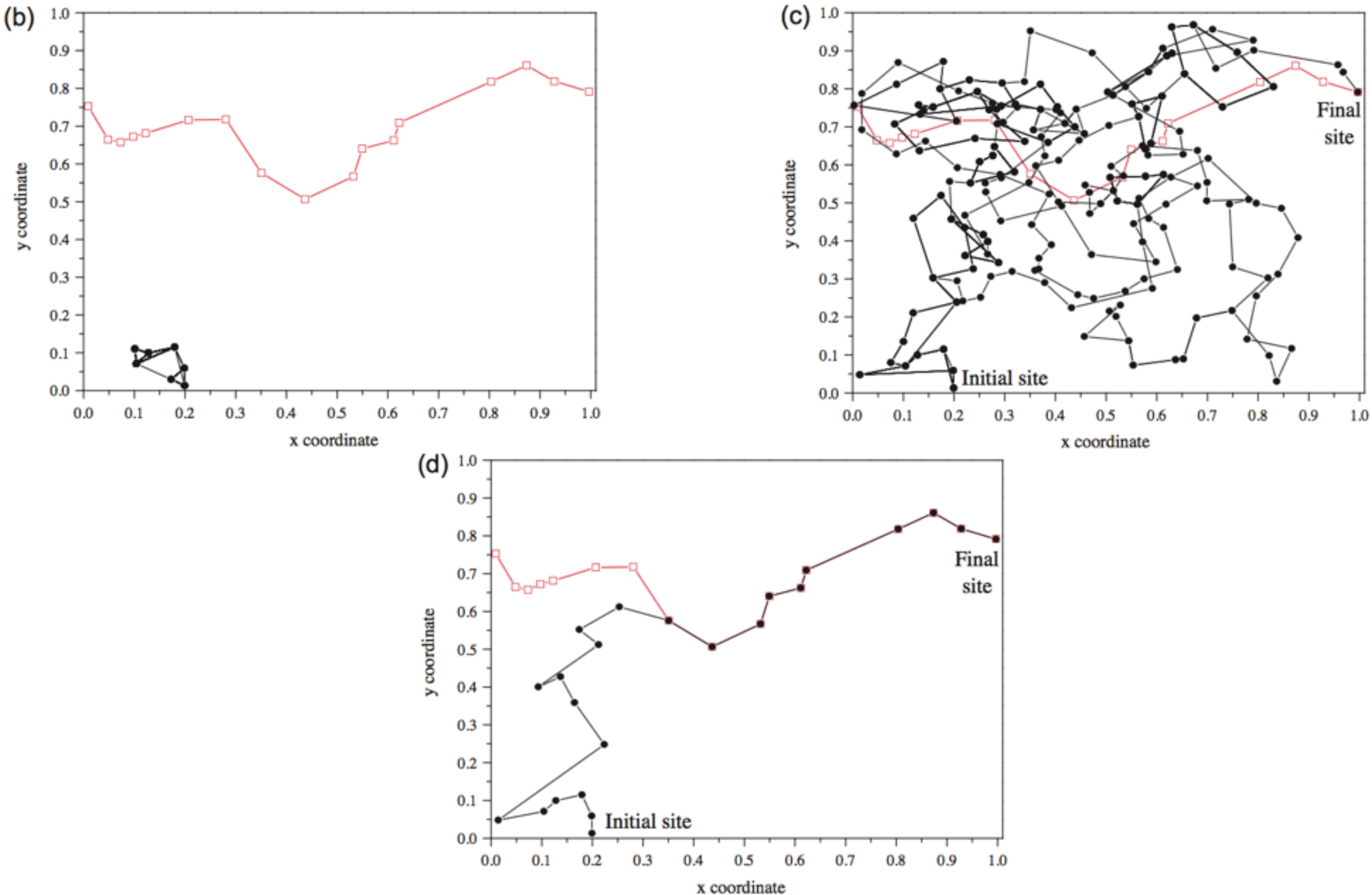
Combining movement decisions and landscape dynamics



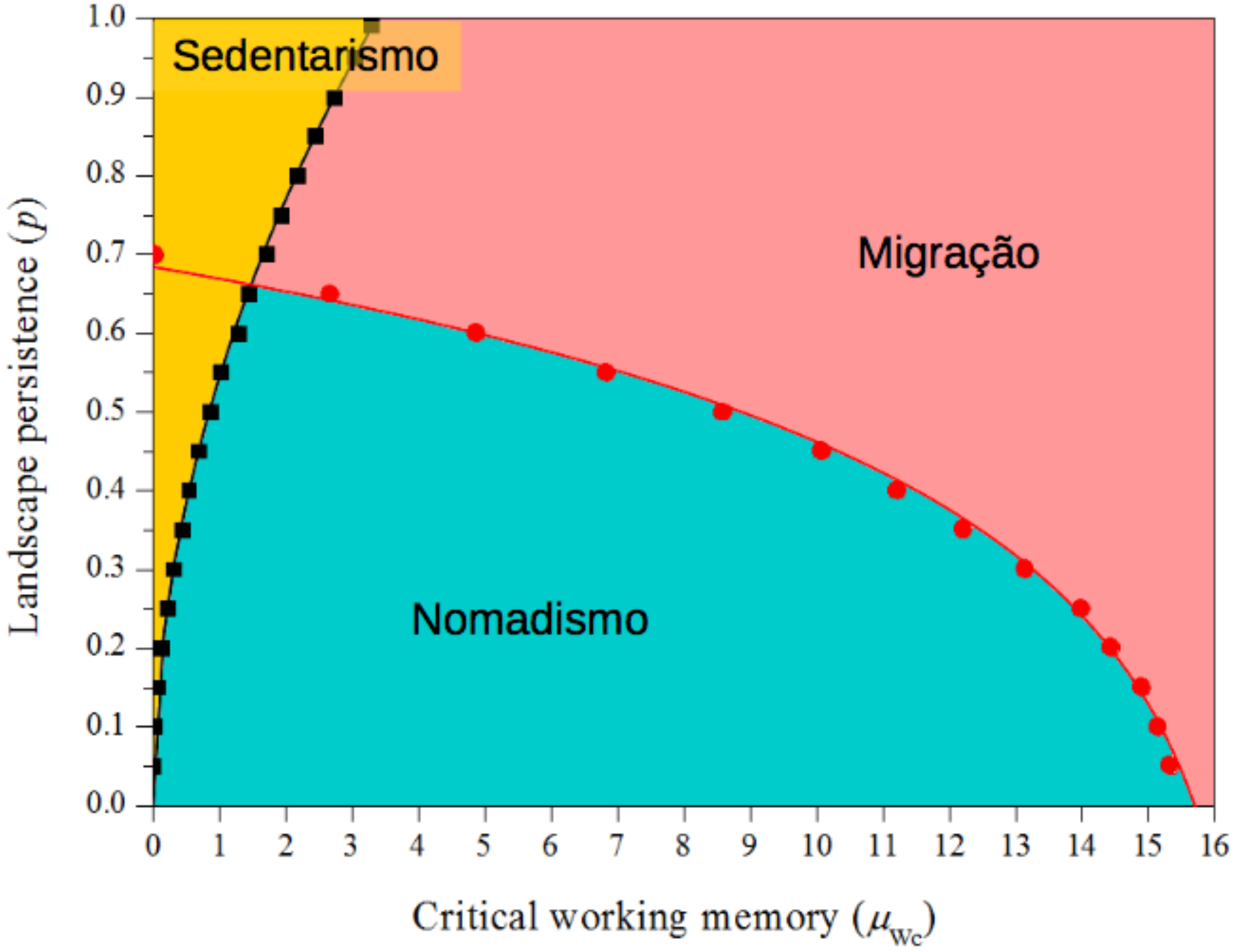
- *Avoid the last visited sites, or*
- *Follow the migration route.*

- *Spatial configuration changes from year to year, according to the persistence level.*

INDIVIDUAL-BASED MODEL



INDIVIDUAL-BASED MODEL



PARTIAL DIFFERENTIAL EQUATIONS MODEL

Combining random searches and individuals spatial memory

$$\begin{cases} \dfrac{\partial w}{\partial t} = \alpha u(1-w) - \dfrac{w}{\mu} \ ; \\[2ex] \dfrac{\partial u}{\partial t} = \underbrace{-\dfrac{\partial}{\partial x}\left[u\left(2M_2\dfrac{\partial}{\partial x}\log(1-w) - M_1\right)\right]}_{\text{memory induced advection}} + \underbrace{M_2\dfrac{\partial^2 u}{\partial x^2}}_{\text{diffusion}} \ . \end{cases}$$

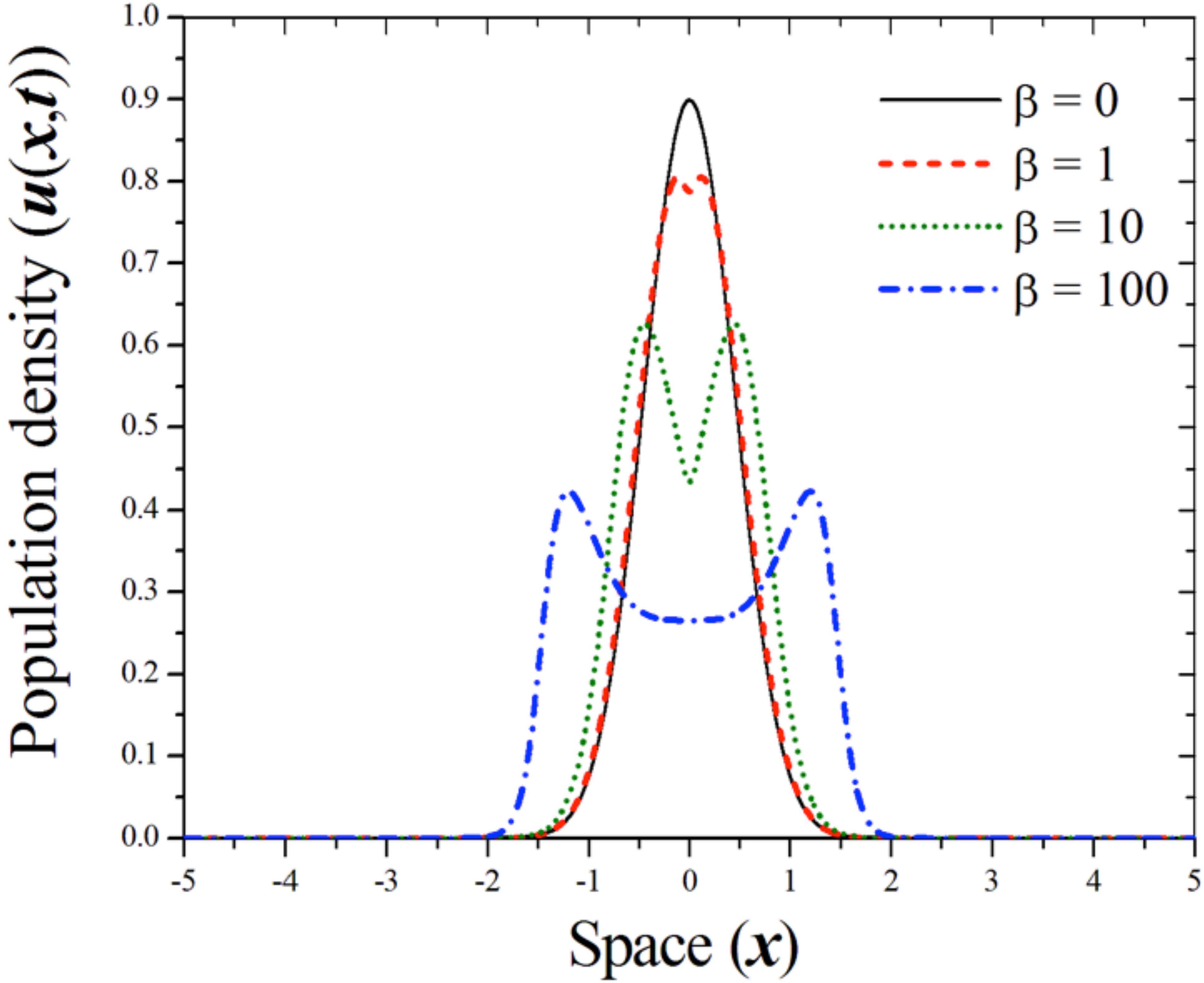
Continuous space and time

PARTIAL DIFFERENTIAL EQUATIONS MODEL

Combining random searches and individuals spatial memory

$$\begin{cases} \dfrac{\partial w}{\partial t} = \beta u(1-w) - w \ ; \\[2ex] \dfrac{\partial u}{\partial t} = -2\dfrac{\partial}{\partial x}\left[u\left(\dfrac{\partial}{\partial x}\log(1-w)\right)\right] + \dfrac{\partial^2 u}{\partial x^2} \ . \end{cases}$$

PARTIAL DIFFERENTIAL EQUATIONS MODEL



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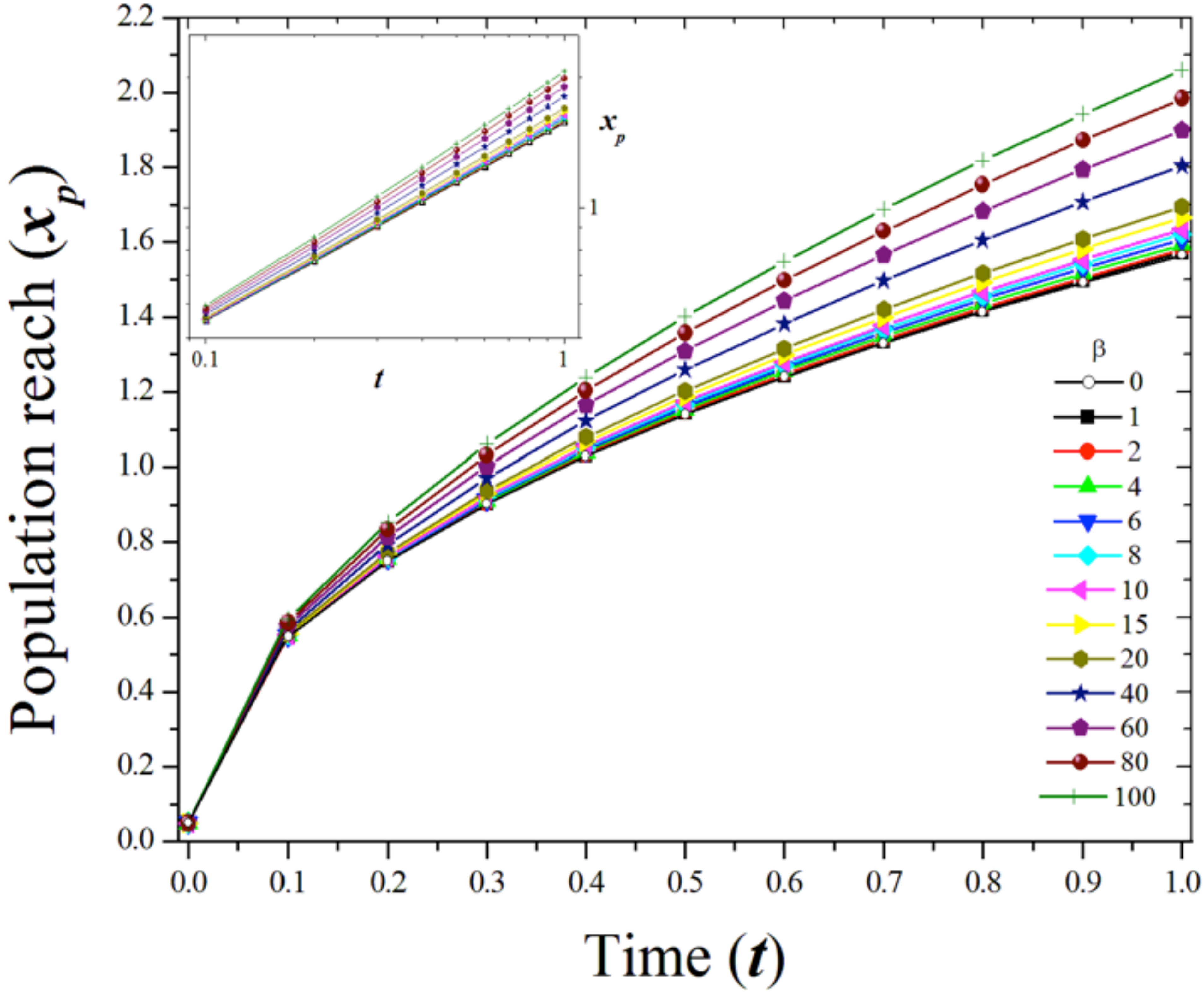

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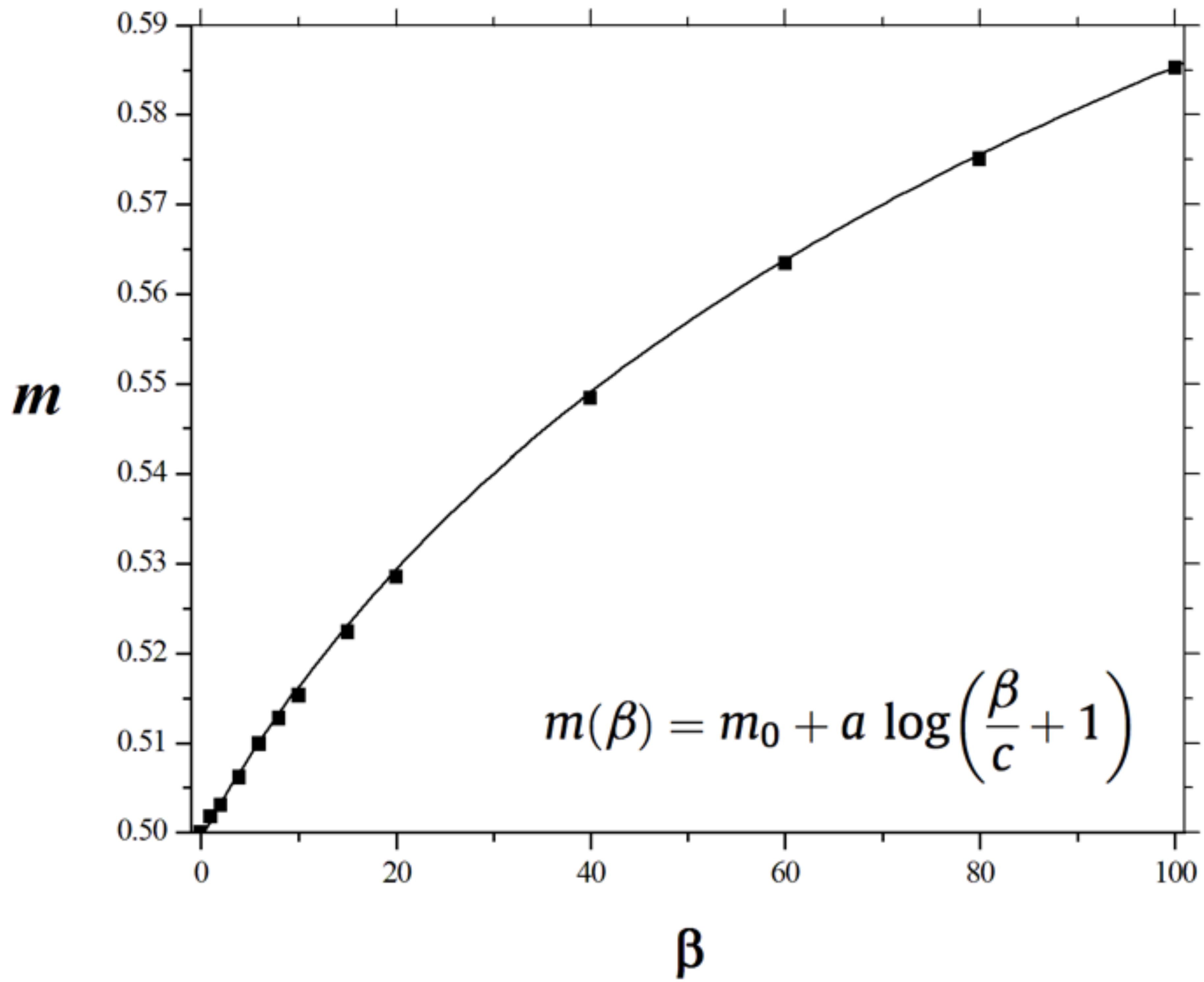

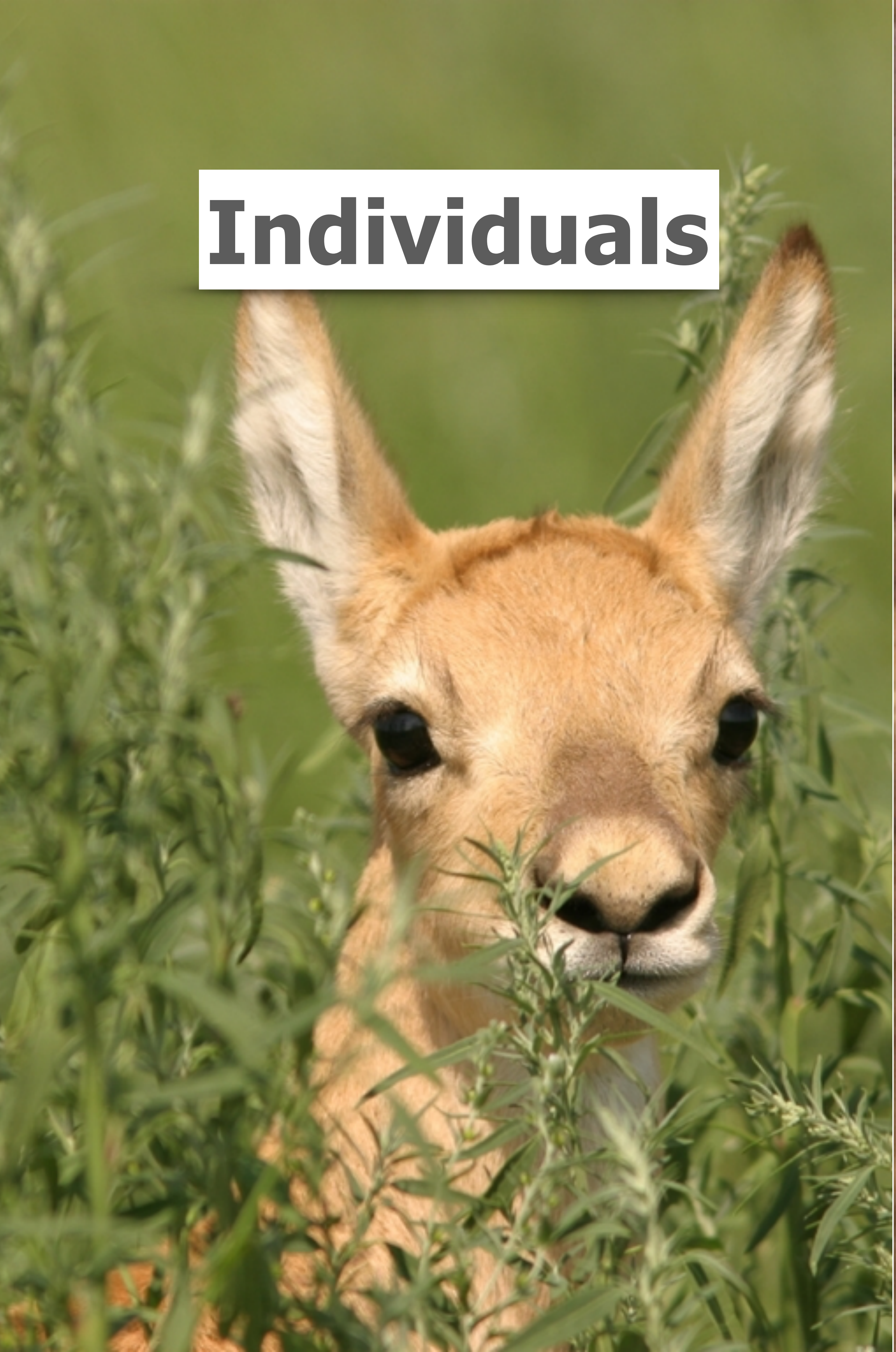
Individuals

Population

POPULATION DYNAMICS

- Births
- Deaths
- Migration
- Resources

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- Births
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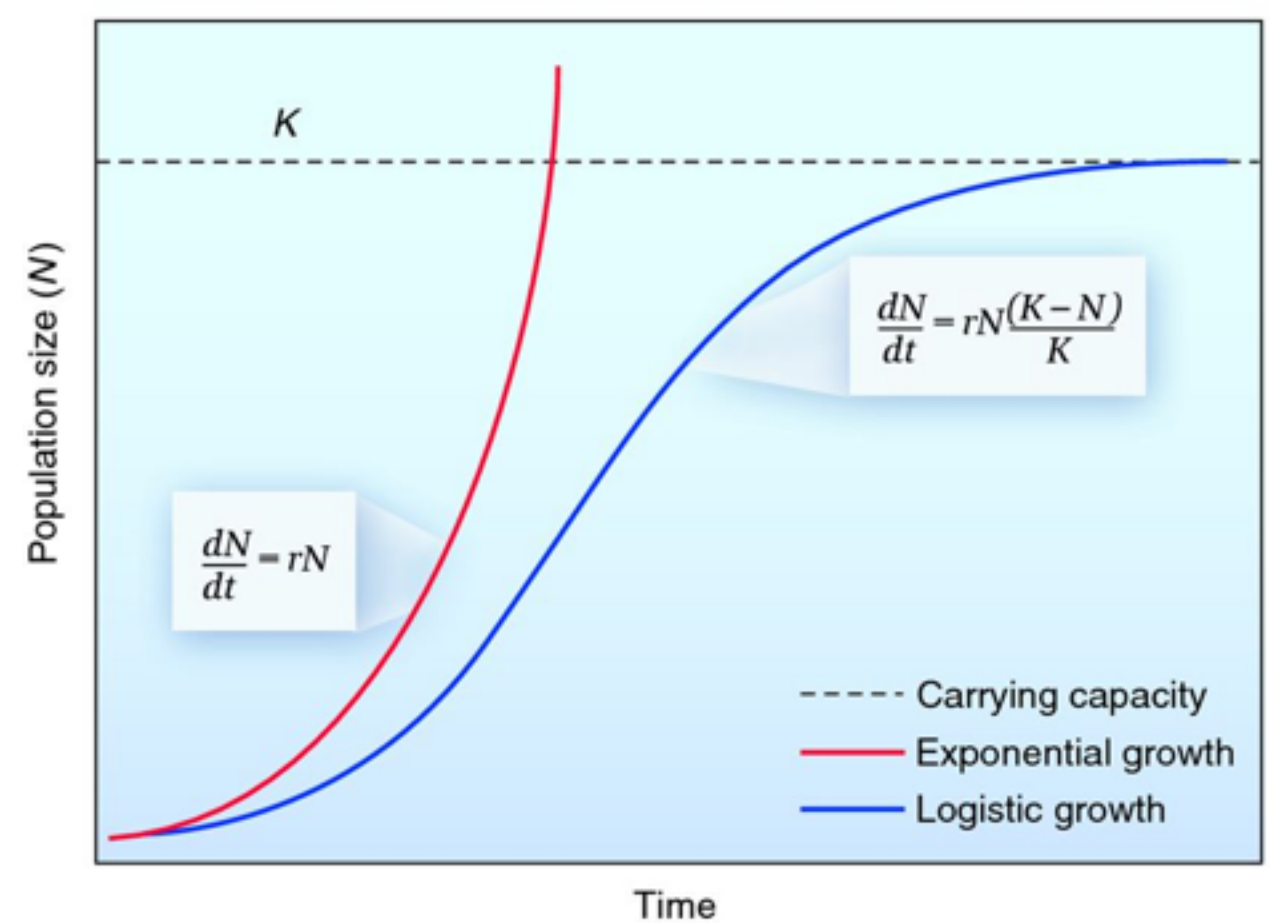
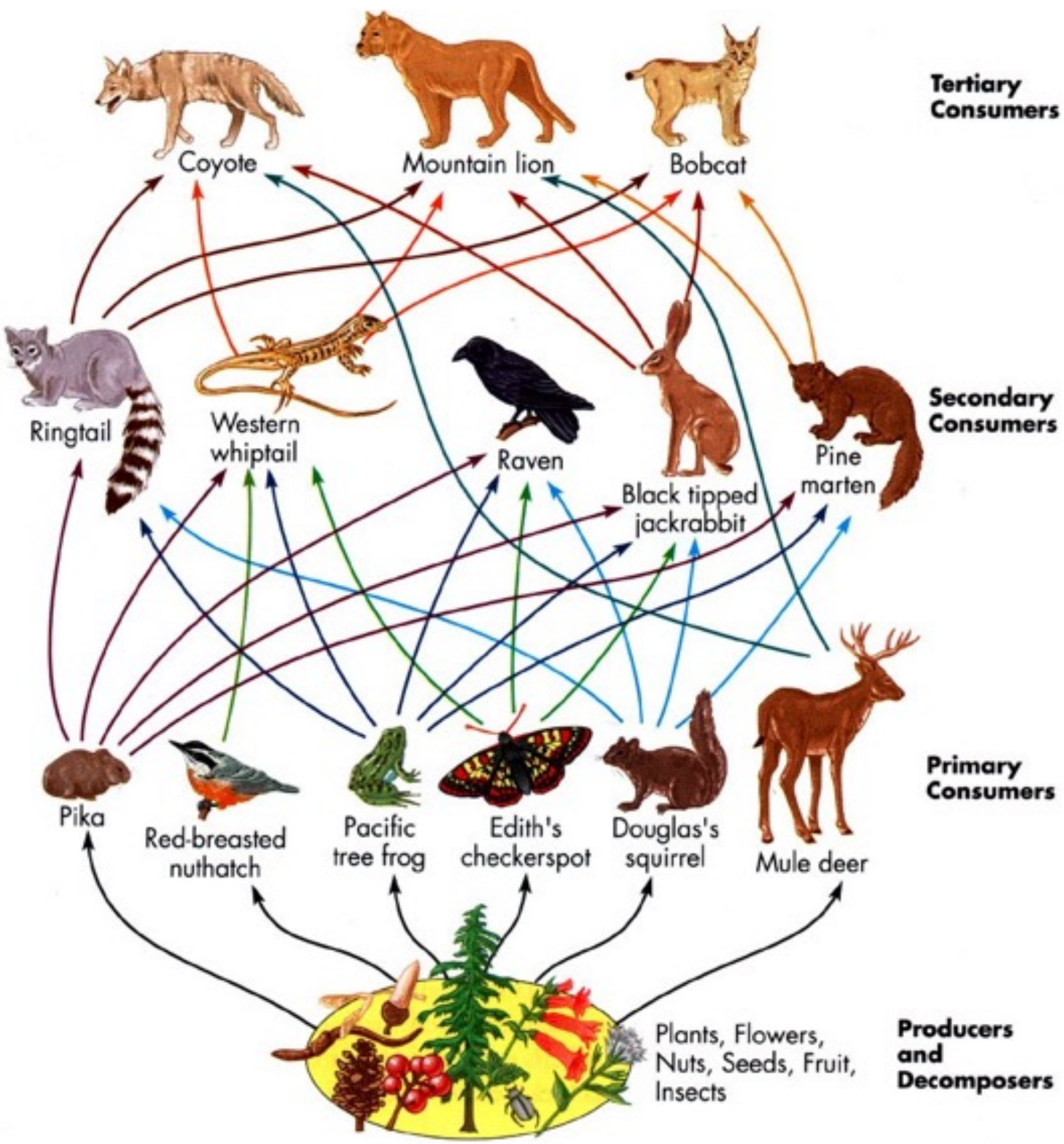


POPULATION DYNAMICS

- Births
- Deaths
- Migration
- Resources

- Structured populations
- Space explicitly modeled
- Habitat loss and fragmentation
- …

COMMUNITIES

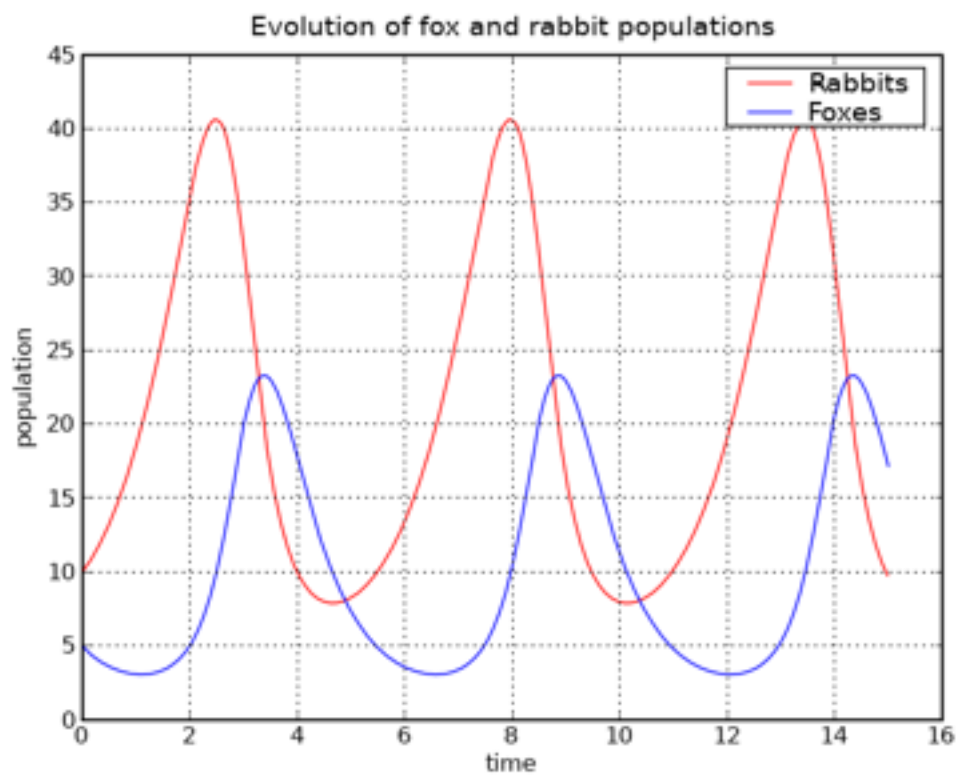


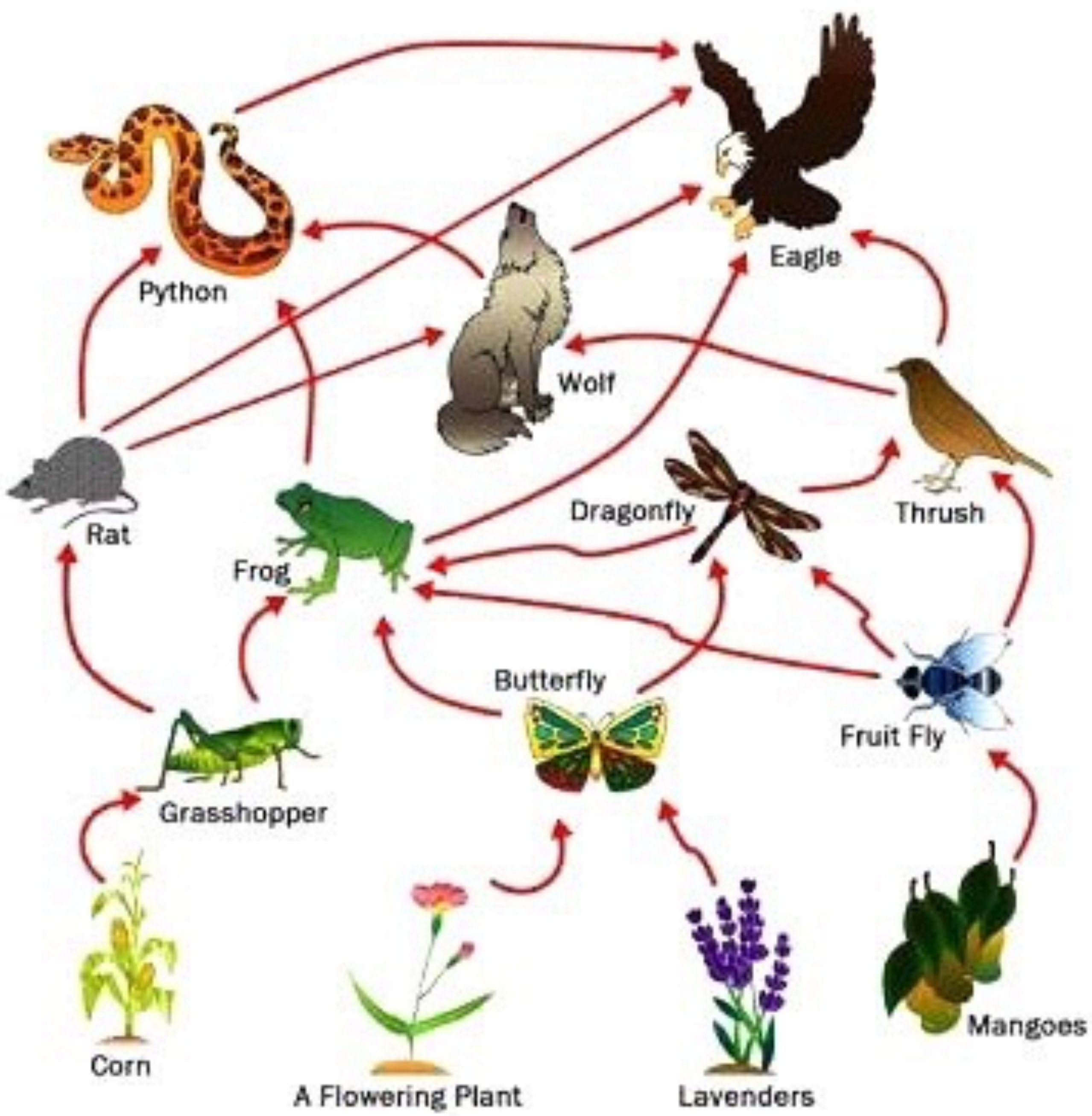
- Interactions
- Resource-consumer
- Competition
- Predation

COMMUNITIES

- Predation

$$\frac{dV}{dt} = \alpha V - \beta V P \ ,$$
$$\frac{dP}{dt} = \delta V P - \gamma P \ .$$





COMMUNITIES

- Interactions
- Resource-consumer
- Competition
- Predation

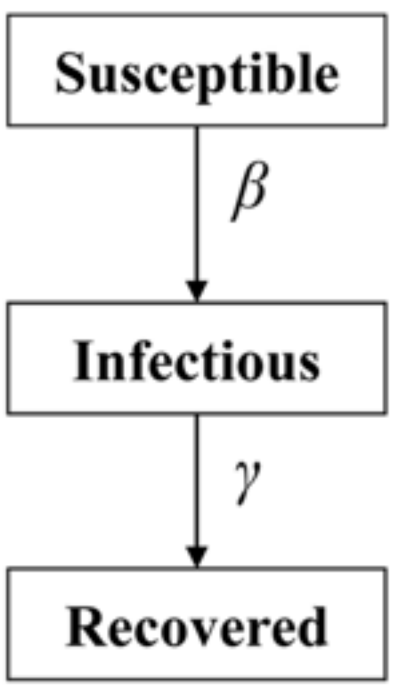
- Structured populations
- Space explicitly modeled
- Handling time
- Habitat loss and fragmentation
- …

EPIDEMICS

- Interaction
- Compartments

EPIDEMICS

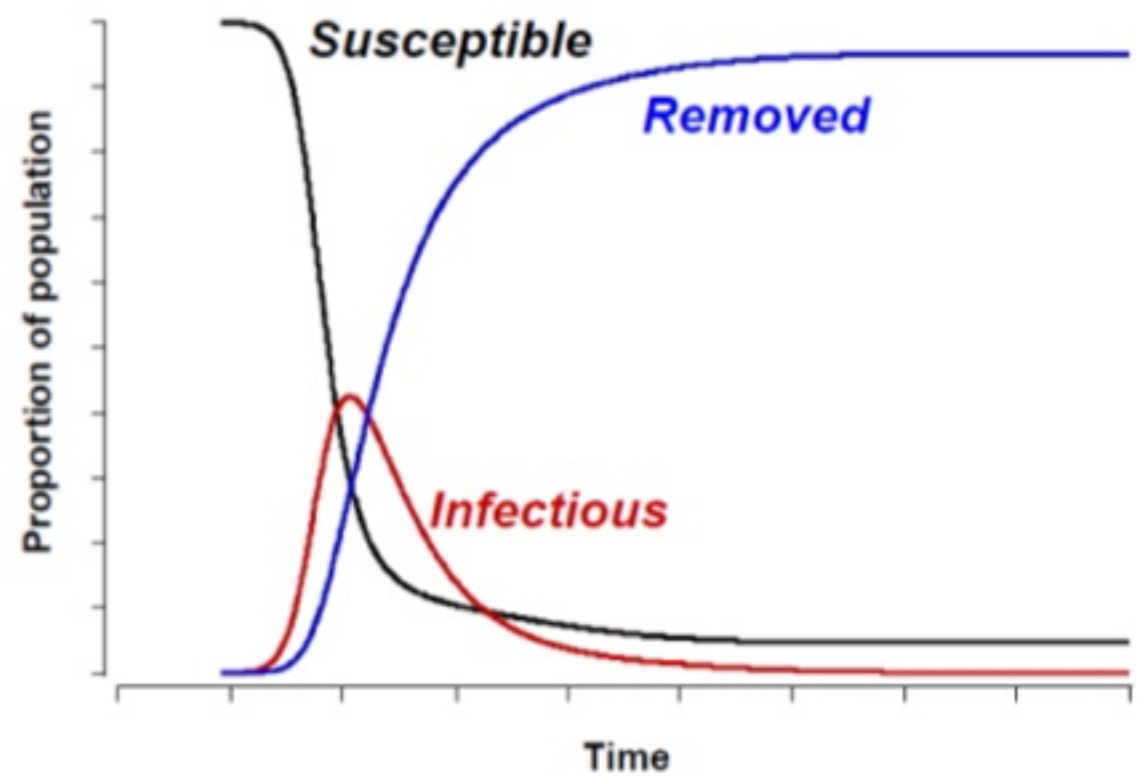
- Interaction
- Compartments



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$



EPIDEMICS

- Interaction
- Compartments

- Structured populations
- Space explicitly modeled
- Vector
- Public Policies
- …

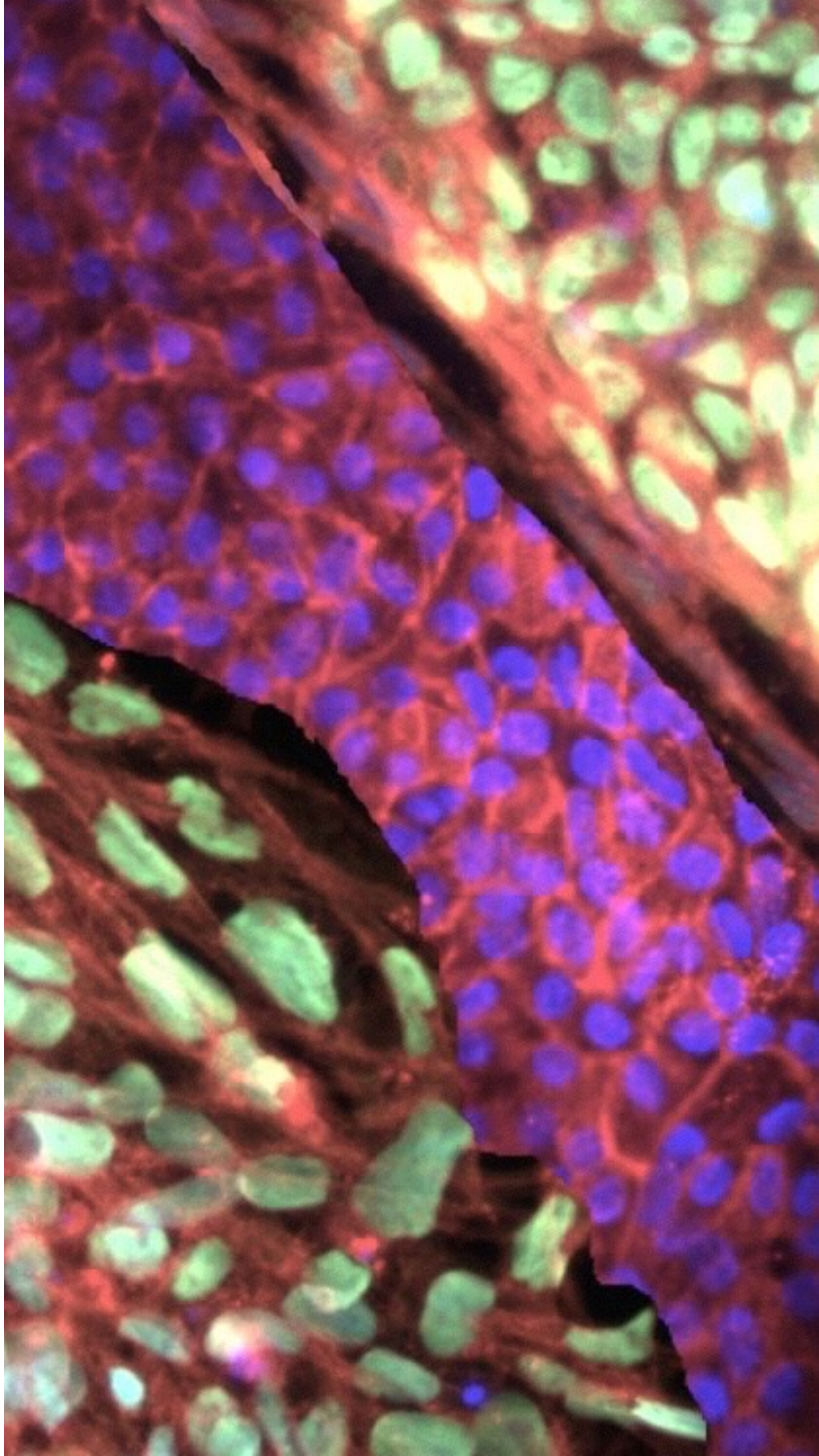
Evolutionary ecology

Cellular and molecular dynamics

Genetics

Tumor growth

Phylogenetic trees



To what extent can we describe the biological world?

Which tools can we use to explore causes, mechanisms and patterns of the biological world? With what degree of specificity and generality?

- *Ran Nathan et al. PNAS 2008;105:19052-19059*
- *Ovaskainen, Knegt, Mar Delgado, 2016, "Quantitative Ecology and Evolutionary Biology Integrating models with data"*
- *Berbert and Fagan, Ecological Complexity 2012 (12) 1-12.*
- *Berbert and Lewis, Ecological Complexity, 2018 (33) 41-48.*
- *Ginzburg et al. Ecological Modelling 2007 (207) 356–362*
- *Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"*
- *Gerda de Vries et al. 2006, "A course in Mathematical Biology"*

Obrigada!