A Simple Approach to Deriving Outlier Labeling Rules for Skewed Distributions

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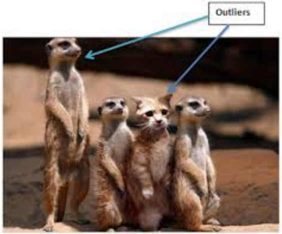
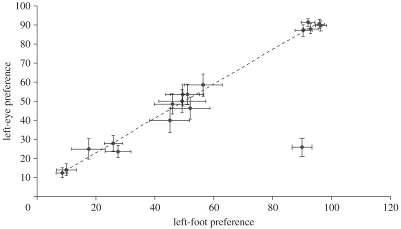
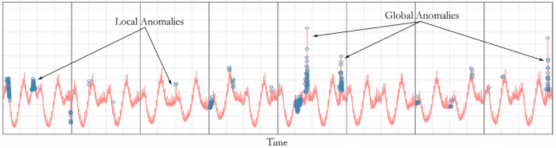
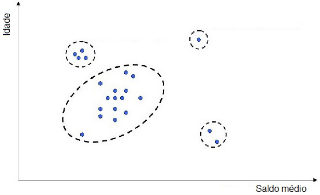
27 de agosto de 2018

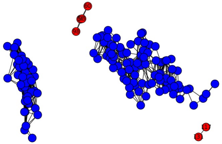
Presence of Abnormality



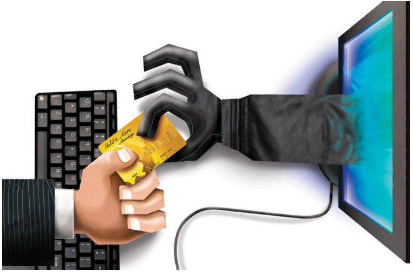








Presence of Frauds [1]



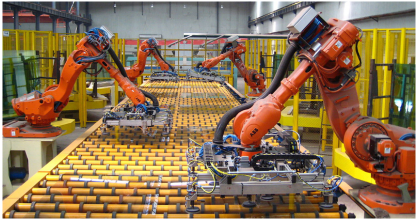
1. Intrusion Detection Systems;
2. Credit Card Fraud;
3. Law Enforcement;
4. Interesting Sensor Events.

[1] www.zoomtech.com.br/fraudes-na-internet-o-que-fazer-se-voce-for-vitima/

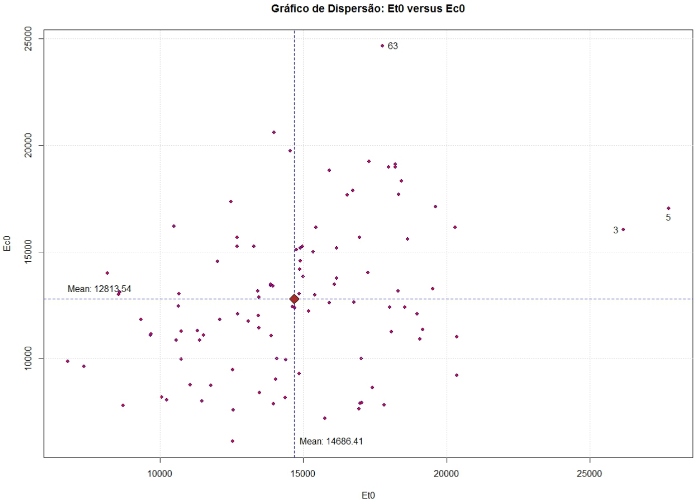
Disease Detection [2]

[2] https://www.heraldsun.com.au/news/victoria/melbourne-scientists-develop-blood-test-for-early-alzheimers-disease-detection/news-story/b61c32f26cf9286e9124f0ea86b21ebb

Human Errors or Mechanical Errors [3]



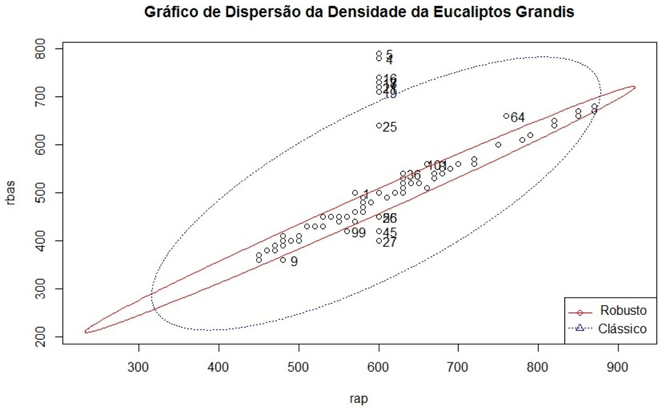
[3] http://www.hojeaprendi.com.br/2014/12/21/encontre-erros-de-digitacao-com-mais-facilidade/



Non-Robust Methods

1. Support Vector Machines - SVM;
2. Maximum Likelihood Estimators - MLE;
3. Least Square Method - LS.

Traditional vs. Robust Estimators



1. Independence;
2. Distribution Gaussian;
3. Absence of Outliers.

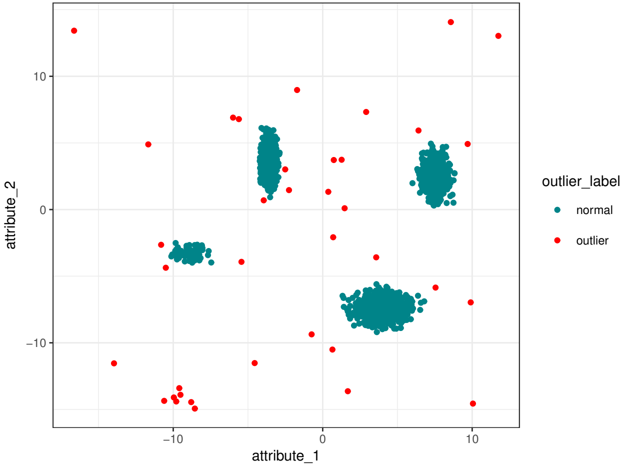
Measure Robustness [4]

Breakdown Point - BP

1. (max breakp. of 0%) $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$
2. (max breakp. of 25%) $\frac{IQR}{a} = \frac{(Q_3 - Q_1)}{a}$
3. (max breakp. of 50%) $MAD = b * med|x_i - med(x_i)|$
4. (max breakp. of 50%) $S_n = c * med_i(med_j|x_i - x_j|)$

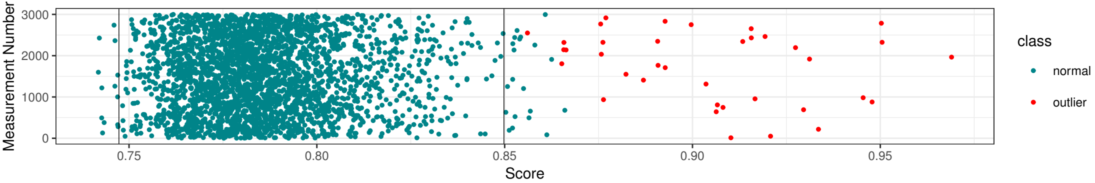
[4] Hampel, F. R. (1974). The influence curve and its role in robust estimation.

Rules for Outliers Detection [5]



[5] http://dx.doi.org/10.7910/DVN/OPQMVF

Rules for Outliers Detection



Model of Location and Scale [6]

Let the cumulative distribution function $F_{\mu,\sigma}$, where F is a continuous function with parameters of location μ and scale σ.

$$\mu \pm (factor)\sigma \tag{1}$$

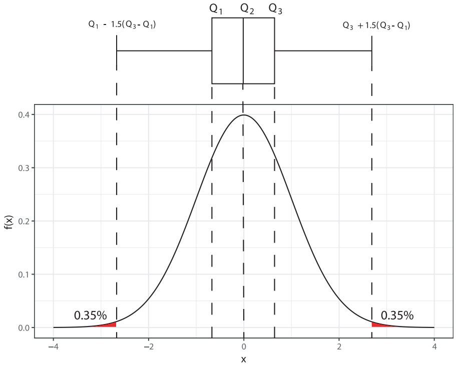
factor = g*k.

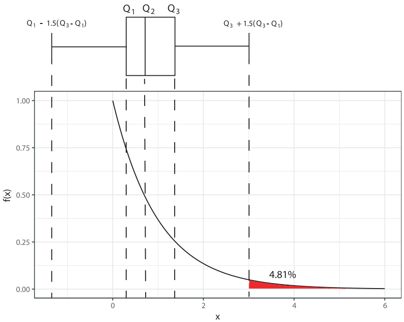
$$k = \frac{F^{-1}(1-\alpha/2) - F^{-1}(0.75)}{F^{-1}(0.75) - F^{-1}(0.25)}. \tag{2}$$

g parameter that consider the size of the sample.
$\alpha = P(X_i \in out(\alpha_n, \mu, \sigma^2) | X_i \notin out(\alpha_n, \mu, \sigma^2))$

[6] Dovoedo, Y. H., and Chakraborti, S. (2015). Boxplot-based outlier detection for the location-scale family.

Classic Boxplot [7]





$$[Q_1 - kIQR; Q_3 + kIQR] \quad (3)$$

$$k = \frac{\phi^{-1}(1-\alpha/2) - \phi^{-1}(0.75)}{\phi^{-1}(0.75) - \phi^{-1}(0.25)} \quad (4)$$

[7] Tukey, J. W. (1977). Exploratory data analysis (Vol. 2).

Adjusted Boxplot [8]

Let $S = \{x_1, x_2, \cdots, x_n\}$ be a sample from a continuous unimodal distribution. The medcouple can be determined as follows:

$$[MC = Med\frac{(x_j - Q_2) - (Q_2 - x_i)}{x_j - x_i},] \quad (5)$$

where Q_2 is the median of S, $x_i \leq Q_2 \leq x_j$ and $x_i \neq x_j$.
If $MC \geq 0$

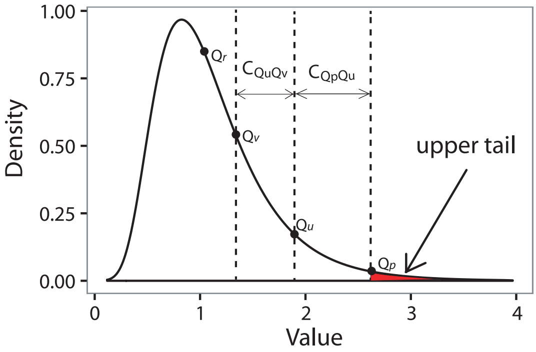
$$[Q_1 - 1.5e^{-4MC}IQR; \quad Q_3 + 1.5e^{3MC}IQR]. \quad (6)$$

If $MC < 0$

$$[Q_1 - 1.5e^{-3MC}IQR; \quad Q_3 + 1.5e^{4MC}IQR]. \quad (7)$$

[8] Hubert, M., and Vandervieren, E. (2008). An adjusted boxplot for skewed distributions.

New Method: Contrast between Quantiles

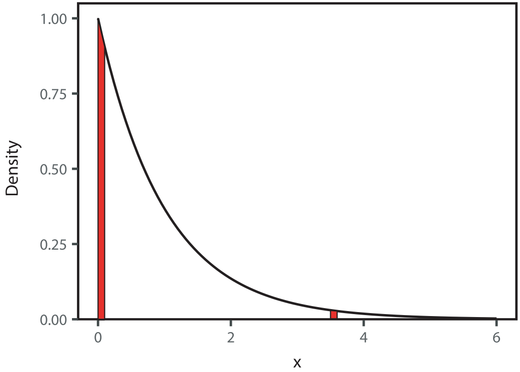


1. Non Parametric;
2. Form Simetrics and Skews;
3. Nominal Outside Rate 0.7%;
4. Scale;

$$[SIQR_L = (Q_2 - Q_1); \quad (8)$$

$$SIQR_U = (Q_3 - Q_2)] \quad (9)$$

5. Contrast between quantiles;
6. Sensitivity constant.



Upper tail

Scenario without contamination

Lower Tail

Scenario with contamination

Tabela: Summary of the outlier rules

Rule	Lower Threshold	Upper Threshold
Boxplot	$Q_1 - 1.5IQR$	$Q_3 + 1.5IQR$
Adj. Boxplot	$\begin{cases} Q_1 - 1.5\,e^{-4MC}IQR & : \quad MC \geq 0 \\ Q_1 - 1.5\,e^{-3MC}IQR & : \quad MC < 0 \end{cases}$	$\begin{cases} Q_3 + 1.5\,e^{3MC}IQR & : \quad MC \geq 0 \\ Q_3 + 1.5\,e^{4MC}IQR & : \quad MC < 0 \end{cases}$
QCR(0.0035)	$\begin{cases} Q_1 - 3\,e^{\left(1.64\left(1/C_{Q_3Q_2}-1\right)\right)}SIQR_L & : \quad C_{Q_3Q_2} \geq 1 \\ Q_1 - 3\,\log_{1.66}\left(1.66/C_{Q_3Q_2}\right)SIQR_L & : \quad C_{Q_3Q_2} < 1 \end{cases}$	$\begin{cases} Q_3 + 3\,\log_{1.66}\left(1.66C_{Q_3Q_2}\right)SIQR_U & : \quad C_{Q_3Q_2} \geq 1 \\ Q_3 + 3\,e^{\left(1.64\left(C_{Q_3Q_2}-1\right)\right)}SIQR_U & : \quad C_{Q_3Q_2} < 1 \end{cases}$
QCR(0.007)	$\begin{cases} Q_1 - 3\,e^{\left(1.64\left(1/C_{Q_3Q_2}-1\right)\right)}SIQR_L & : \quad C_{Q_3Q_2} \geq 1 \\ Q_1 - 3\,\log_{2.1}\left(2.1/C_{Q_3Q_2}\right)SIQR_L & : \quad C_{Q_3Q_2} < 1 \end{cases}$	$\begin{cases} Q_3 + 3\,\log_{2.1}\left(2.1C_{Q_3Q_2}\right)SIQR_U & : \quad C_{Q_3Q_2} \geq 1 \\ Q_3 + 3\,e^{\left(1.64\left(C_{Q_3Q_2}-1\right)\right)}SIQR_U & : \quad C_{Q_3Q_2} < 1 \end{cases}$

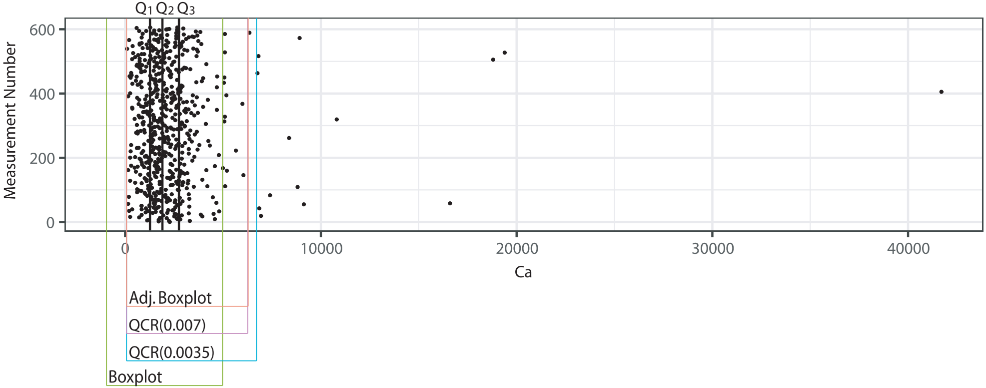
Tabela: Average outside rates determined by both upper and lower thresholds of each considered rule for uncontaminated samples of size 10,000.

Distribution	MC	Boxplot	Adj. Boxplot	QCR(0.0035)	QCR(0.007)
$exp(1)$	0.333 ± 0.001	0.0481 ± 2e-04	0.00288 ± 6e-05	0.00353 ± 9e-05	0.0070 ± 1e-04
$\Gamma(0.1, 0.5)$	0.505 ± 0.001	0.0756 ± 2e-04	0.00191 ± 6e-05	0.00349 ± 8e-05	0.0078 ± 1e-04
$\Gamma(0.1, 0.75)$	0.396 ± 0.001	0.0579 ± 2e-04	0.00243 ± 6e-05	0.00347 ± 8e-05	0.0073 ± 1e-04
$\Gamma(0.1, 1.25)$	0.292 ± 0.001	0.0419 ± 2e-04	0.00320 ± 9e-05	0.0036 ± 1e-04	0.0068 ± 1e-04
$\Gamma(0.1, 5)$	0.136 ± 0.001	0.0193 ± 2e-04	0.0062 ± 1e-04	0.00398 ± 9e-05	0.0060 ± 1e-04
$N(0, 1)$	0.000 ± 0.001	0.0070 ± 1e-04	0.0073 ± 1e-04	0.0072 ± 1e-04	0.0074 ± 1e-04
χ^2_1	0.505 ± 0.001	0.0756 ± 3e-04	0.00192 ± 5e-05	0.00350 ± 8e-05	0.0078 ± 1e-04
χ^2_5	0.197 ± 0.001	0.0279 ± 2e-04	0.0041 ± 1e-04	0.0037 ± 1e-04	0.0062 ± 1e-04
χ^2_{20}	0.095 ± 0.001	0.0139 ± 2e-04	0.0074 ± 1e-04	0.0051 ± 1e-04	0.0066 ± 1e-04
$F(90, 10)$	0.259 ± 0.001	0.0516 ± 2e-04	0.0222 ± 3e-04	0.0128 ± 1e-04	0.0177 ± 2e-04
$F(10, 10)$	0.274 ± 0.001	0.0535 ± 2e-04	0.0194 ± 3e-04	0.0119 ± 1e-04	0.0170 ± 2e-04
$F(10, 90)$	0.155 ± 0.001	0.0238 ± 2e-04	0.0076 ± 1e-04	0.0048 ± 1e-04	0.0072 ± 1e-04
$F(80, 80)$	0.099 ± 0.001	0.0163 ± 1e-04	0.0100 ± 1e-04	0.0071 ± 1e-04	0.0089 ± 1e-04

Note: MC is the medcouple - robust measure of skewness

C-horizon of the Kola Data

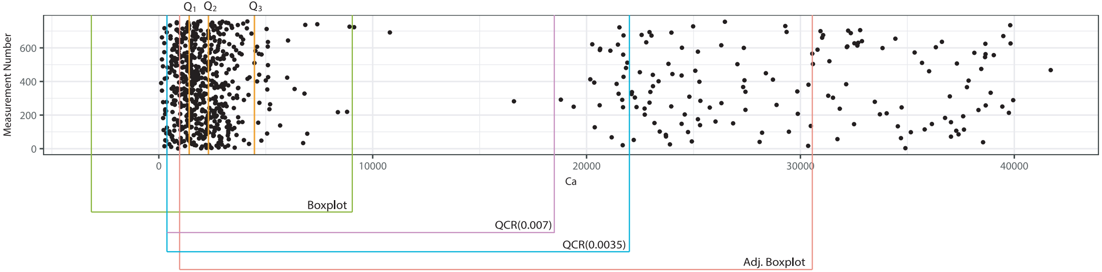
C-horizon data of 606 observations collected in the Kola Project (1993-1998, Geological Surveys of Finland and Norway and Central Kola Expedition, Russia [9])



[9] Reimann, C., and Garrett, R. G. (2005). Geochemical background—concept and reality.

C-horizon of the Kola Data

20% contamination.



Conclusions

1. High Performance for more than 15% outliers;
2. It is describe well skewed;
4. its computational complexity is $O(n)$.
5. It does not need of simulation to chance "outside rate".

References

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2 Brys, G., Hubert, M., & Struyf, A. (2004). A robust measure of skewness. Journal of Computational and Graphical Statistics, 13(4), 996-1017.

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