# Implementation of Gauss-Newton method with complex variables for the estimation Electrical Impedance Tomography images

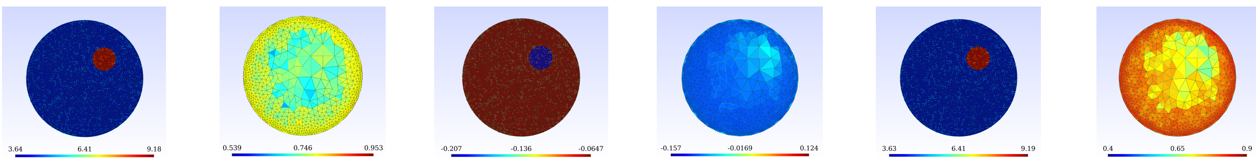
**André Juan Ferreira Martins de Moraes***, **Olavo Luppi Silva***
*UFABC, Santo André, Brasil
e-mail: olavo.luppi@ufabc.edu.br

**Introduction:** The Electrical Impedance Tomography (EIT) is a relatively new technique applied to medical imaging systems. It is regarded as extremely promising, specially in the study of the muscular system, as presented on recent studies [1]. The image is obtained through the solution of the so-called *inverse problem*, which results in the estimation of the electrical impeditivity distribution of the domain under analysis. This is done by a computer model of the problem, whose implementation was the main objective of this project. A *numerical phantom* was used to obtain simulated measurements of electrical potential necessary for solving the inverse problem.

**Methods:** An extensive literature review was performed covering (i) the theoretical basis of inverse problems [2]; (ii) complex variables and its applications; and (iii) optimization methods applied to EIT. This was followed by the computer implementation of Gauss-Newton method according to [1] and [2], written in C, using the mathematical library GSL and the software package ATLAS. The software Gmsh was used to create and visualize the finite element meshes. The estimated image of impeditivity distribution $\zeta(x, y, z)$ is related to conductivity $\sigma$, relative permittivity $\epsilon_r$ and frequency $\omega$ according to Eq. 1.

$$\zeta = \frac{\sigma}{\sigma^2 - (\omega\epsilon_r\epsilon_0)^2} - j\left(\frac{\omega\epsilon_r\epsilon_0}{\sigma^2 - (\omega\epsilon_r\epsilon_0)^2}\right) \tag{1}$$

**Results:** Figures 1a, 1c and 1e show the distributions of resistivity ($\rho = Re(\zeta)$), reactivity ($\chi = Im(\zeta)$) and absolute value of impeditivity ($\zeta = \rho + j\chi$) assigned to a mesh with 45528 tetrahedral elements used to obtain simulated measurements. An electrical conductivity of $0.275\,11\,\mathrm{S\,m^{-1}}$ and a relative permittivity of 4425 was assigned to most part of the domain. To the circle, at 2nd quadrant, was assigned a conductivity of $0.109\,01\,\mathrm{S\,m^{-1}}$ and a relative permittivity of 2224. The result obtained after 50 iterations of the algorithm, with a mesh of 6941 elements, is shown at Figures 1b, 1d and 1f. It is clear that the circular object is detected but not perfectly reconstructed. Additionally the diameter of the object is overestimated in Fig. 1b, 1d and 1f due to the presence of large elements in the inner portion of the mesh used in the reconstruction.




(a) (b) (c) (d) (e) (f)

Figure 1 – Resistivity of: (a) numeric phantom, (b) EIT estimation. Reactivity of: (c) numeric phantom, (d) EIT estimation. Impeditivity of: (e) numeric phantom, (f) EIT estimation.

**Conclusion:** The analysis of the results confirms that the developed algorithm in this project was successfully implemented. EIT images were obtained within the degree of precision expected for the employed method. However, there's room for improvements of the algorithm and its applications in broader contexts.

**References:** [1] O. L. SILVA. Muscle contraction detection using Electrical Impedance Tomography. PhD thesis, Escola Politécnica da Universidade de São Paulo, 2012. [2] P. J. VAUHKONEN. Image Reconstruction in Three-Dimensional Electrical Impedance Tomography. PhD thesis, Department of Applied Physics, University of Kuopio, Finland, 2004.