Adaptive Self-information cancellation for Analog Network Coding: A Preliminary Evaluation

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Abstract - *The Analog Network Coding allows the exchange of information between two nodes in a two-way relay channel network in half of the time when compared to a time division multiplexing scheme. However, the user nodes need to know the involved channels impulse responses to perform detection at the received signal, and, therefore, channel estimation techniques are necessary. Therefore, this work proposes a new way to perform the self-information cancellation adaptively and jointly estimating one of the channel impulse responses. It is also shown that the proposed system performs well, in terms of BER, when compared to the performance of the same system with perfect knowledge of all channel responses.*

Palavras-chave: *Analog Network Coding; Channel Estimation; Adaptive Filtering.*

Introduction

The Two-way Relay Channel (TWRC) has gained renewed interest with the advent of Physical-layer Network Coding (PNC) technique [1] applied in wireless communication systems. In the TWRC, two nodes, called user nodes, exchange information using a relay node. If the system uses time division multiple access (TDMA), the nodes can exchange information using four time intervals, as shown in Figure 1: 1st - node 1 transmits data to node R (relay); 2nd - node R transmits the information received from node 1 to node 2; 3rd - node 2 transmits data to node R; 4th - node R transmits the data received from node 2 to node 1.

Another possibility would be the use of the frequency division multiplexing (FDM) technique,

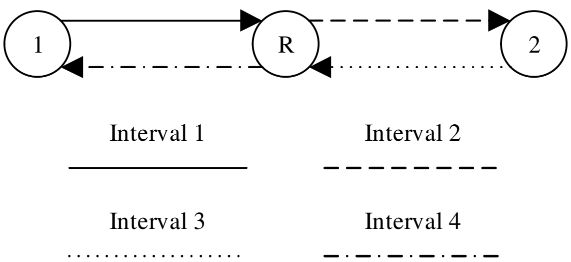

Figure 1 – TDMA applied to the TWRC.

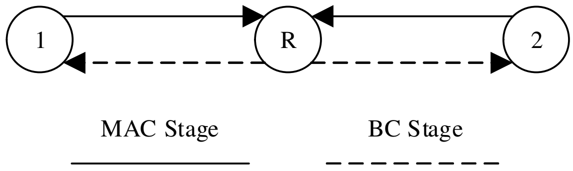

Figure 2 – PNC applied to the TWRC.

where both users could transmit to the relay at the same time, but in different frequencies.

The PNC technique allows both users to transmit to the relay node at the same time and same frequency, taking advantage of the interference that happens at the wireless medium. Using PNC, the number of time intervals required to exchange information between nodes 1 and 2 reduces to two, as shown in Figure 2: 1st - the multiple access (MAC) phase, where the user nodes send their information simultaneously to the relay node, and 2nd - the broadcast (BC) phase, where the relay sends its information to both user nodes.

The relay node needs to perform some operations over the received signal so that the user nodes can decode the information sent by the other user node from the signal transmitted by the relay. This process is known as PNC mapping. For a binary-phase shift keying (BPSK) modulation, a possible PNC mapping is shown in [1], and for M-ary quadrature amplitude modulation (M-QAM) the PNC mapping is developed in [2].

Another kind of PNC mapping is the Analog Network Coding (ANC) [3] , which allows the relay to just amplify the signal received during the MAC phase and forward it to the user nodes. This scheme is also called *amplify-and-forward*-PNC (AF-PNC), and allows any type of modulation.

The PNC and ANC techniques is commonly applied in wireless networks, and, therefore, the channel effects in the transmitted signals needs to be taken into consideration in order to allow the user nodes to perform the detection of the information [4]. Thus, it is important for the receivers to obtain the channel state information (CSI), what can be done by appropriate channel estimation techniques. The work in [5] considers flat-fading channels to develop the Maximum Likelihood and the Linear Maximum Signal-to-Noise Ratio estimators, together with an optimal training sequence to reduce the estimator mean squared error (MSE). For ANC systems with orthogonal frequency division multiplexing (OFDM) modulation, a Least Squares (LS) estimators, either by using an OFDM symbol as training sequence, or by using pilot sub-carriers, is proposed in [6]. The linear minimum mean squared error (LMMSE) estimator for OFDM-PNC is developed in [7]. A channel estimator in frequency domain for OFDM-PNC is proposed in [8], by using the cyclic property of the discrete Fourier transform, which is used to implement OFDM systems.

Most of these works considers a time invariant channel or that it changes in a slow rate. Thus, it is interesting to develop algorithms that consider a time variant channel in PNC systems. The current work takes advantage of the way the self-information is removed from the signal received by the user node to develop an adaptive channel estimation and self-information extraction. It is important to mention that this work is in a preliminary stage, and the goal is to present the main idea and some preliminary results.

System Model and Problem Formulation

Due to the symmetry of the TWRC network, the base-band discrete time signal model is developed for node 1, since it will be equivalent for node 2.

Let $\mathbf{x}_i$ be the vector containing the N BPSK symbols $x_{i,n}$ sent by user node i at instant n and $\mathbf{X}_i$ its convolution matrix. The signal received by the relay node can be written as:

$$\mathbf{y} = \mathbf{X}_1\mathbf{h}_{1R} + \mathbf{X}_2\mathbf{h}_{2R} + \mathbf{w}_R, \tag{1}$$

where $\mathbf{h}_{iR} = [h_{iR,0}\ h_{iR,1}\ \cdots\ h_{iR,L-1}]^{\mathrm{T}}$ is the vector containing the L taps of the channel between the node i and the relay, and $\mathbf{w}_R = [w_{R,0}\ w_{R,1} \cdots w_{R,N-1}]$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ_w^2 ($\mathcal{N}(0, \sigma_w^2)$).

The relay will amplify the received signals by a factor α and transmit the resulting signal. Then, the signal received by node 1 can be written as:

$$\mathbf{y}_1 = \alpha\mathbf{X}_1\mathbf{a} + \alpha\mathbf{X}_2\mathbf{b} + \mathbf{w}, \tag{2}$$

where $\mathbf{a}$ is the vector containing the samples of the cascaded channel resulted from the convolution between $\mathbf{h}_{1R}$ and $\mathbf{h}_{R1}$ and $\mathbf{b}$ is the channel obtained

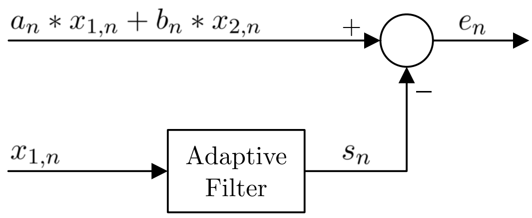

Figure 3 – Adaptive self-information extraction system.

from the convolution between $\mathbf{h}_{2R}$ and $\mathbf{h}_{R1}$, where $\mathbf{h}_{R1}$ is the vector containing L taps of the channel between the relay and node 1 during the BC phase. The noise component is defined as $\mathbf{w} = [w_0\ w_1\ \cdots\ w_{N-1}]$ where $w_n = h_{1,n} * w_{R,n} + w_{1,n}$, and $w_{1,n}$ is AWGN ($\mathcal{N}(0, \sigma_w^2)$) at node 1 at instant n.

Detection of the information sent by node 2 can be performed at node 1 as:

$$\hat{\mathbf{x}}_2 = \arg\min_{\mathbf{x}_2} |\mathbf{y}_1 - \alpha\mathbf{X}_1\mathbf{a} - \alpha\mathbf{X}_2\mathbf{b}|^2. \tag{3}$$

A practical way of performing this at node 1 is to use a two-steps approach. In the first step, node 1 extracts its self-information from $\mathbf{y}_1$, as shown in (4). Note that, to perform this task correctly, the knowledge of $\mathbf{a}$ is necessary.

$$\begin{aligned}\tilde{\mathbf{x}}_2 &= \mathbf{y}_1 - \alpha\mathbf{X}_1\mathbf{a} \\ &= \alpha\mathbf{X}_2\mathbf{b} + \mathbf{w}.\end{aligned} \tag{4}$$

Then, in the second step, the signal $\tilde{\mathbf{x}}_2$ needs to be equalized. Linear equalizers such as Zero-forcing (ZF) or algorithms such as the Maximum Likelihood Sequence Estimation (MLSE) can be used for this purpose [9].

In this work, it is shown that the self-information cancellation, represented by equation (4) can be viewed as an echo cancellation problem and, therefore, can be implemented as an adaptive algorithm. In order to do this, consider the system shown in Figure 3, where there is no noise influence, for the sake of simplicity.

Let s_n be the samples at the filter output at instant n, and the defining the error signal e as

$$e_n = a_n * x_{1,n} + b_n * x_{2,n} - s_n, \tag{5}$$

the adaptive filter can minimize the MSE or, in other words, minimize the following cost function:

$$\begin{aligned}J(n) &= \mathrm{E}\left[|e^2(n)|\right] \\ &= \mathrm{E}\left[(a_n * x_{1,n} + b_n * x_{2,n} - s_n)^2\right].\end{aligned} \tag{6}$$

Equation (6) can be rewritten as:

$$\begin{aligned} J(n) = {} & \mathrm{E}\left[(b_n * x_{2,n})^2\right] \\ & + 2\,\mathrm{E}\left[(b_n * x_{2,n})(a_n * x_{1,n} - s_n)\right] \\ & + \mathrm{E}\left[(a_n * x_{1,n} - s_n)^2\right]. \end{aligned} \tag{7}$$

The channels taps h_{iR} and h_{R1} are independent complex Gaussian random variables with zero mean and unity variance. It is worth noting that, in this work, the channels are considered time invariant.

As the cascaded channels are independent from the transmitted information and each of their coefficients has zero mean, it is easy to show that the term $\mathrm{E}\left[(b_n * x_{2,n})(a_n * x_{1,n} - s_n)\right] = 0$. Then, (7) becomes:

$$\begin{aligned} J(n) = {} & \mathrm{E}\left[(b_n * x_{2,n})^2\right] \\ & + \mathrm{E}\left[(a_n * x_{1,n} - s_n)^2\right]. \end{aligned} \tag{8}$$

The adaptive filter does not have control on the term $\mathrm{E}\left[(b_n * x_{2,n})^2\right]$, since it only affects the term s_n. Thus, minimizing $J(n)$ means minimizing the term $\mathrm{E}\left[(a_n * x_{1,n} - s_n)^2\right]$. As it is a squared error, minimize this term means that it approximates to zero. Hence, the error signal e_n can be considered a good estimation of $b_n * x_{2,n}$. This way, by equalizing the error signal with respect to the channel b, it is possible to obtain the information sent by node 2. This system is feasible once the node 1 has the knowledge of x_1, its own transmitted signal.

Simulations and Results

The first simulation evaluates the convergence of the adaptive filter in the proposed system considering no noise effects. The algorithm to update the filter taps used in this preliminary evaluation is the *Least Mean Squared* (LMS) [10] algorithm. As shown in the previous section, the error used to adapt the filter will actually approximate $b_n * x_{2,n}$. Hence, to better visualize the convergence of the algorithm, a different error will be used. This error will be defined as

$$\tilde{e}_n = b_n * x_{2,n} - e_n. \tag{9}$$

Therefore, the closer e_n is to $b_n * x_{2,n}$, the smaller is $\tilde{e}_n$. Averaging the MSE from 1000 realizations, each one simulating the transmission of 10000 symbols, the convergence of the $\tilde{e}_n$ is shown in Figure 4. The channels h_{iR} and h_{R1} were generated with $L = 5$ taps, so channels **a** and **b** have 9 taps each. To adapt the filter with 9 taps, the adaptation step used is $\mu = 0.001$. It is possible to see that the

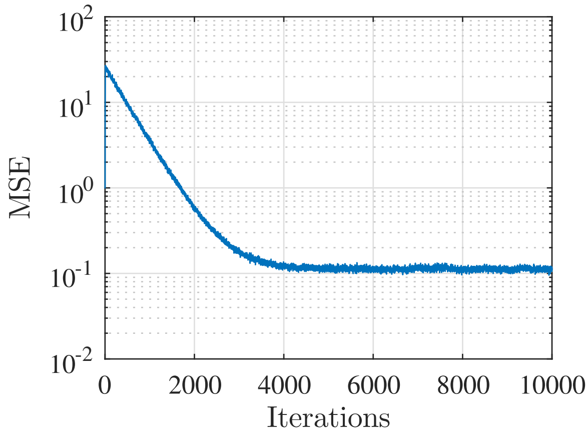

Figure 4 – Evaluation of LMS convergence considering the error $\tilde{e}_n$.

error achieves the floor in about 4000 iterations, with an MSE of 10^{-1} dB.

The proposed system was, then, simulated in a noisy scenario. The bit error rate (BER) at node 1 was obtained through the transmission of 10^7 bits and is shown in Figure 5, using the same parameters from the previous simulation. Furthermore, the channel **b** is supposed to be perfectly know by node 1. It is important to highlight that the CSI curve in Figure 5 assumes perfect knowledge of both channels **a** and **b** and that the self-information cancellation is performed by (4).

Initially, it is transmitted a training sequence containing 5000 random BPSK symbols. This is done in order to allow the adaptive filter to achieve convergence before transmitting information symbols. It is important to notice here that for the filter taps update, it is only necessary the knowledge of $x_{1,n}$, i.e, the information sent by node 1 itself, and the received signal. Therefore, once the filter achieves steady-state, it is not necessary to transmit training sequences anymore. This is the main advantage of the proposed system.

It is possible to see that for low SNR values, there is no performance difference between the proposed system and the system that uses perfect CSI. A slight difference can be seen for high SNR values. Therefore, the usage of the proposed system is feasible in terms of BER performance.

Conclusions

In this work, an adaptive self-information cancellation technique applied in ANC systems is proposed. It can be seen that the proposed technique operates well in term of BER and it is shown that its advantage over conventional schemes is the possibility of

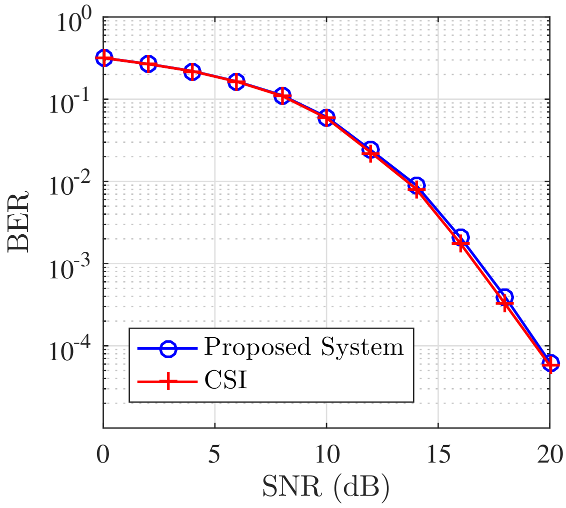

Figure 5 – BER comparison between the proposed scheme and the one using equation (4) with perfect CSI.

update the filter taps even during the transmission of information symbols.

As future works, the system will be evaluated in a scenario with time-varying channels and with other adaptive filtering algorithms, such as the recursive least squares (RLS) [10].

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