Generalized Likelihood Ratio Test for slope analysis in seismic data processing

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Abstract – *The purpose of this work is to employ a high resolution technique based on Generalized Likelihood Ratio Test to substitute the semblance criterion in the search for locally coherent events in slope analysis. The aim is to generate a high resolution spectra to promote a reliable and intuitive event detection scheme. The proposed method makes use of a binary hypothesis test to detect locally coherent events in slope analysis. The proposed detector, a Generalized Likelihood Ratio Test, has the relevant advantage that the definition of the detection threshold is independent of the quality of the data. The performance of the method is presented through processing results of real seismic data, which indicates a significant improvement in vertical resolution.*

Keywords: *seismic signal processing, semblance, coherence analysis, seismic tomography.*

Introduction

In seismic processing, the picking process is quite common in many processing flows. One of the most common usage is in velocity analysis. In the last years, the exploration of high resolution techniques have been proposed in the literature. For example, in [1] expressive results were found using a type of detector called GLRT (*Generalized Likelihood Ratio Test*) [2] for velocity analysis.

Another case of interest is in slope analysis required, for instance, in some types of tomography, like stereotomography. Stereotomography [3,4] was proposed in 1998 with the main idea of creating a velocity macromodel using slope tomography. One of the advantages of the method is to deal with relatively complex geological structures without restriction on lateral velocity variations. Moreover, it does not depend on sensible and costly PreStack Migration cycles. The main characteristic of the stereotomography approach is that it does not require a stratigraphic model as it considers only locally coherent events. Therefore, there is no need that the events represent a wave reflection in continuous interfaces. The selected data, especially slope/slowness information, are used as reference for the tomographic inversion.

The purpose of this work is to extend the results found in [1], using a high resolution technique to substitute the usage of semblance in the search for locally coherent events in slope analysis. The aim is to generate high resolution spectra to promote a reliable and intuitive event detection scheme.

Undoubtedly, one great challenge to this tomography approach has been a consistent automatic picking of events from prestack data [3,4,5,6]. For more details see the original work [4].

Proposed Method

To pick representative events, a widespread solution is based on the semblance [7] coherency measure. Similar to the conventional velocity analysis in the CMP (*Common MidPoint*) gather, the slopes estimation can be done in the CS (*Common Shot*), CR (*Common Receiver*) and CO (*Common Offset*) gathers. Although for picking selection for in stereotomography just two gathers is enough [4], a more robust automatic picking is achieved with 3 or 4 gathers.

The procedure can be viewed as a fitting of gather traces to the traveltime model, which is given by $t(t_0, \Delta x) = t_0 + p_x \Delta x$, where t_0 is the time instant under analysis, $\Delta x = x - x_0$ denotes the displacement in the x axis with respect to the central reference trace and $p_x = \sin(\phi)/v$ denotes the slowness (slope) of the event, in which ϕ is the emergence angle at the surface and v is the surface velocity. Thus, after the semblance computation for the desired range of slope p_x at each time sample of that reference trace, the result is a slope spectrum, which is used as the main information for data picking.

In stereotomography, the picking of each event is made on CS and CR gathers simultaneously (or any combination in pair of CS, CR and CO [5], which are associated by the coordinate of the reference trace under analysis.

A natural way to support the picking process is to set a threshold in the coherence spectrum, such that above this value an event is detected. One great problem of this approach is the dependence of some prior knowledge of the data quality to define a good value for the threshold. Moreover, in the deepest regions the reflections tend to present smaller density of energy requiring different values of threshold.

One way to overcome these difficulties is to look at the problem as a hypothesis test in which the threshold can be uniformly defined all over the coherence volume through a probabilistic value. Here, the definition of the threshold value is not only intuitive, but also independent of the quality of the data.

Similar with the idea proposed in [1] for velocity analysis, it was developed a method based on follow hypothesis model for seismic event detection:

$$\mathcal{H}_0: \mathbf{x}_k = w_0 \mathbf{n}_k \quad (1)$$
$$\mathcal{H}_1: \mathbf{x}_k = A\mathbf{s}_k + w_1 \mathbf{n}_k \quad (2)$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote respectively the null and alternative hypothesis, $\mathbf{x}_k \in \mathbb{R}^{N_s}$ is a snapshot vector with data samples of N_s sensors in the instant k, for $k = 1, \dots, N_t$, $\mathbf{n}_k$ represents noise and interference with scale factor w_0 and w_1 for both null and alternative hypothesis, respectively. Here, it is assumed that the vectors $\mathbf{s}_k \in \mathbb{R}^{N_s}$ are known and represent the waveform of the signal/event on the select window. In the other hand, the amplitude of the event - given by the parameter A is unknown.

Hence, the PDF (*Probability Density Function*) of $\mathcal{H}_1$ cannot be determined, and the optimal detector, called Neymam-Pearson [2] detector, is unfeasible. Despite that, since the signal component in $\mathcal{H}_1$ is deterministic and the type of PDF of the noise can be determined through a long tail distribution estimation, it is possible to project suboptimal detectors. That is why GLRT detectors are used. One strong characteristic of the proposed GLRT detector is the that it is CFAR (*Constant False Alarm Ratio*), as the probability of false alarm can be kept constant independent of the noise power, i.e., the threshold of the likelihood ratio adapts to the data. For this reason, the detector is suitable to operate equally, without any change in the detection threshold, in both shallow and deep portions of the data.

Adopting the linear model for the hypotheses in (1) and (2), assuming $\mathbf{s}_k = \mathbf{1}$ and incorporating the Laplacian distribution for the noise $\mathbf{n}_k$, we can design a GLRT for the two-sided parameter test problem:

$$\mathcal{H}_0: A = 0; \quad (3)$$
$$\mathcal{H}_1: A \neq 0; \quad (4)$$

with $-\infty \leq A \leq \infty$ and data set given by the set $\{\mathbf{x}_k\}_{k=1}^{N_t}$. To make the detector completely independent from the data, we suppose that the scale factor w_0 of $\mathcal{H}_0$ is unknown.

The asymptotic detection performance of the GLRT is given by a χ_1^2 (central Chi-squared PDF with 1 degree of freedom) under $\mathcal{H}_0$ and a $\chi'^2_1(\lambda)$ (noncentral Chi-squared PDF with 1 degree of freedom and noncentrality parameter λ) under $\mathcal{H}_1$, with $\lambda = 2NA^2/\sigma^2$ and $\sigma = \sqrt{2}w_1$ (see, *e.g.*, [2,8]). As the number of samples $N = N_s N_t$ is generally large for seismic data, the asymptotic PDFs for the test statistics are really representative. This approximation facilitates the computation of the threshold.

Results

To compare the results of conventional slope analysis using semblance and the proposed detector, two datasets of Brazilian basins are used: *i*) Jequitinhonha offshore basin; *ii*) Tacutu onshore basin.

Figure 1 shows the slope analysis of two different CS (Common Shot) sections of the Jequitinhonha data. Traditional semblance is shown on Figures 1.a and 1c and GLRT on Figures 1.b and 1.d. Both coherence measurements use a scale between 0 and 1, where values close to 1 show better confidentiality of the existence of events in each analyzed slope. The results for the GLRT are shown in terms of the probability of detection, which is given by the CDF (*Cumulative Distribution Function*) of the $\chi'^2_1(\lambda)$ random variable that approximates the statistics of the hypothesis test, when evaluated with the maximum likelihood estimates of A and σ (see, *e.g.*, [2,8]).

It is important to observe that semblance only evaluates energy information, while the GLRT gives information of the whole PDF of the given hypotheses. Thus, the GLRT spectrum is more suitable for a unique limiar definition for the whole dataset, since it is independent from the data quality.

Considering Figure 1, it is possible to observe better vertical resolution, *e.g.* around 4.5 and 5.1 seconds, and also an increase in the definition of the first arrival, around 2.9 seconds. It is important to note that the lack of vertical resolution using the semblance in Figure 1.a increases the difficulty of building a reliable automatic picking. In the other hand, the GLRT detector (Figure 1.b) proves to give better vertical resolution, highlighting the distinction of events that are close in time. Similar effects can be seen in Figures 1.c and 1.d.

The resolution issue becomes more evident when the data has low quality, more common in onshore data. Similar with the example above, Figure 2 shows a slope analysis for two different CS sections for the Tatucu onshore basin, Brazil. For example, on Figure 2.b, on times 1.8, 2.3, and on Figure 2.d, on times 1.2, 2.3 and close to 3.8 seconds it is possible to see a good contrast in the two metrics. Notice that semblance (Figures 2.a and 2.c) do not evaluate correctly the data. Since the used travel time is exactly the same, the difference is due to additional information of the PDF used by GLRT.

Another advantage, especially with the increasing of time, is the usage of only one value of threshold in whole data. On Figures 2.d, close to 3.8 this effect is quite clear. While in the semblance panel it is very difficult to find one slope event, GLRT shows a coherence as high as on the events found on the shallower area.

Conclusion

We proposed a GLRT detector for slope analysis, aiming the improvement of resolution of the slope spectrum when compared to the approach that uses the semblance criterion. Processing results of onshore and offshore seismic data has shown a significant enhancement of vertical resolution even for the deepest regions. Another noteworthy characteristic is that the method allows the definition of a unique detection threshold for picking processes, whose value is independent of the noise power.

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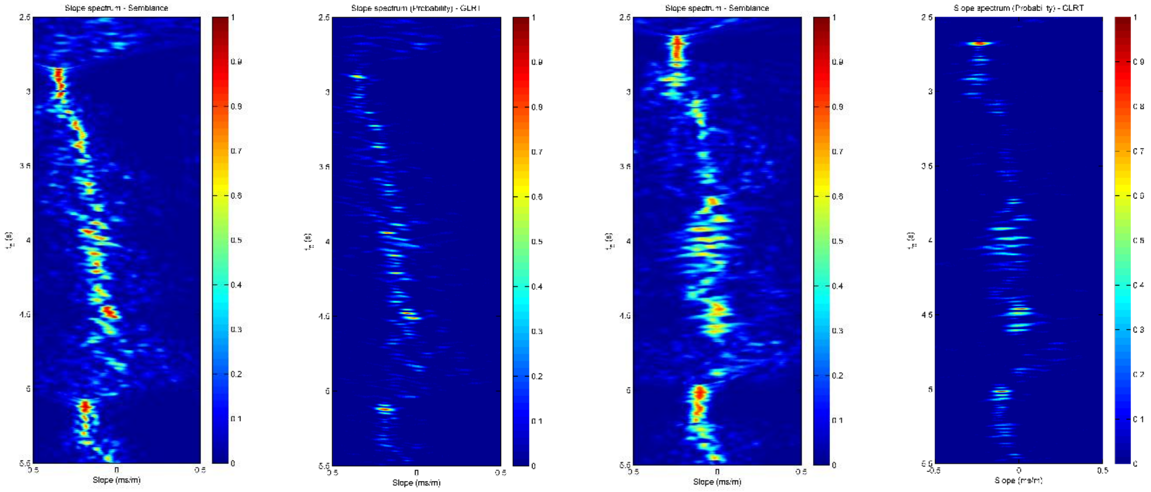
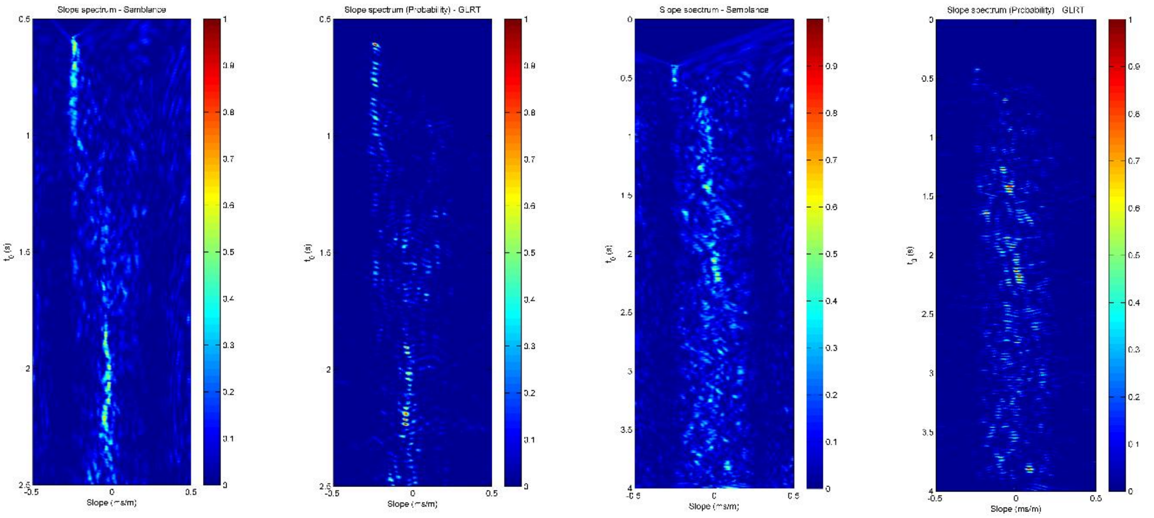


(a) (b) (c) (d)

Figure 1. Two different CS sections spectrum for slope analysis from Jequtinhonha Offshore Basin: (a) Semblance *S*; (b) Detection Probability with GLRT of the first example; and (c) Semblance *S*; (d) Detection Probability with GLRT of the second example.



(a) (b) (c) (d)

Figure 2. Two different CS sections spectrum for slope analysis from Tacutu Onshore Basin: (a) Semblance *S*; (b) Detection Probability with GLRT of the first example; and (c) Semblance *S*; (d) Detection Probability with GLRT of the second example.